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Strategy for Teaching Numerical Expressions through the Wordwall Platform for EJA Students

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Abstract

This work is a pedagogical practice with students of Youth and Adult Education of a public school in Brazil with insertion of the basic operations of mathematics and numerical expressions. The objective of this research was to analyze the contributions of Gamification in meaningful learning based on the Theory of Didactic Situations with the contents of basic operations and numerical expressions with EJA high school students. The methodology at work is the qualitative approach with descriptive-exploratory objectives, applied to twenty-three EJA students, is composed of dynamic parts that occur simultaneously, following the stages of concept, structuring, application, development, resolution and conclusion of each student from the Didactic Contract and Didactic Transposition inserted in the concept of arithmetic and numerical expressions. Results show that gamification plays an important role in teaching mathematics learning inserted in the EJA high school class, which can be demonstrated at the time of the calculations described in the proposed activity. Finally, it is observed that the study carried out with Gamification and the students' discourse had a positive impact on the learning of all students, especially with regard to basic operations and numerical expressions.

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INTRODUCTION

In mathematics education, several modifications of traditional teaching are carried out with the use of Digital Information and Communication Technologies (DICT), in particular, Gamification, an active methodology that has grown rapidly in several educational institutions, with the aim of encouraging and motivating student action, maintaining the attention of participants and promoting learning.

The arrival of increasingly advanced technologies has allowed new ways of doing interactive activities, readapting the traditional teaching method, with the teacher including in the classroom the new teaching practices with the use of Digital Technology (DT), in addition to enhancing new ways of organizing and creating learning situations through interactive games according to school planning.

It is important to emphasize that gamification acquires active methodology contours when the teacher provides an interactive environment with clear rules and allows students to feel in a position of developing skills (Moran, 2015). Thus, games have four basic elements: voluntariness, rules, objectives and feedbacks.

Silva, Sales, and Castro (2019), describe that voluntariness is the acceptance of rules, objectives and feedbacks - the objectives indicate the direction of the participant in the concentration and dedication to achieve the goals proposed in the game - the rules develop during the game and the behavior of the players - and the feedbacks are the communication of the performance and postures informed to the participants.

Given this scenario, it is possible to note that, in the educational context, the realization of activities with games can lead students to a meaningful and fun interaction. This represents a strategy that goes beyond traditional teaching barriers, pointing to new methods and visions regarding what it is to learn and teach mathematics.

Gamification, among the contextualized educational options, is a promising active methodology, which includes students of Youth and Adult Education (EJA) who, in their learning, are motivated by the use of mobile technologies in various mathematics contents.

Moran (2004; 2013), mobile digital technologies aim to make educational institutions transform traditional teaching, in which the teacher is the center of knowledge, for meaningful and participatory learning, with face-to-face interactions and other activities with technological resources, maintaining social and personal bonds, being together virtually and expanding the concept of teaching and learning in the temporal field, where knowledge is allowed by the collaborative action between teacher and student.

Thus, having experience with the use of technological resources in teaching and learning, we propose to use the "Numerical Expression" Gamification of the WordWall repository, to create situations that provide knowledge in an investigative way of the concepts of the four operations (addition, subtraction, multiplication and division) using parentheses, brackets and keys for the EJA class of High School Night, of a public institution, in the municipality of Quixeramobim, Ceará, Brazil.

EJA follows a model that is followed by the Secretary of Education (SEDUC) of the State of Ceará. This means that throughout the year, subjects are changed and new ideas are taught. This includes new ways of teaching, such as using digital technologies and educational materials.

The present work is the result of studies and discussions regarding the appropriate use of technological resources in mathematics teaching, which occurred during the development of technical training with teachers from the state network of Ceará. According to Tardif (2007), the teacher cannot work alone on his professional knowledge, that is, it is necessary to establish a partnership with the students for the inclusion of didactic materials available at that interactive moment in the classroom.

This study, of a qualitative and exploratory nature, involved the investigation of a group of students (research subjects), from a comprehensive public educational institution, enrolled in EJA High School (Basic Education). Given the fact that students with different levels of knowledge present in the EJA class are able to solve problems using different mathematical methods, including the mathematical symbols (keys, brackets and parentheses) that are present in the operations of basic mathematics.

When considering the application of Gamification at different levels of mathematics education and in various areas of teaching, this study set out to question: what are the advantages of Gamification to assess the meaningful learning of EJA high school students, based on the Theory of Didactic Situations (TSD), and the contents of basic mathematical operations and numerical expressions?

The objective of the research was to analyze how the WordWall tool can help EJA students, based on TSD, with the contents of basic mathematical operations and numerical expressions.

Theoretical Reference

The word "gamification" has been included since 2002, but it only gained notoriety in 2010, with the Technology, Entertainment, Design (TED) conferences and the Khan Academy platform, which have publicized and inserted new strategies around gamified and interactive activities (Teichner & Fortunato, 2015).

In this context, some authors of the French Didactics of Mathematics were included, who present research and studies focused on the teaching of mathematics, based on the theories of Brousseau (1997) and Chevallard (1991), which address epistemological phenomena such as Didactic Contract, Didactic Situations and Didactic Transposition.

Pais (2001) describes that the Didactics of Mathematics is a trend in mathematics education, whose focus is the structuring of concepts and theories, comparative to the educational specifications of mathematical knowledge in school, which causes strong links with the formation of mathematical concepts at the experimental level of the teacher's pedagogical practice and in the theoretical concept of academic research.

Based on this perspective, Chevallard (1991) defines the "Didactic Transposition", which was first used by the French sociologist Michel Verret in his doctoral thesis Le temps des étides in 1975, which he defines as "a content of knowledge, having been designated as knowledge to teach then undergoes a set of adaptive transformations that will make it fit to take place among the objects of teaching" (Chevallard, 1991, p. 45).

Lorenzato (2006) states that the use of manipulable Didactic Material (MD) in the teaching of Mathematics is crucial for the teaching and learning process, and it is essential that the teacher properly uses the didactic materials, exploring the concepts of each topic addressed. Explanations do not have the same effectiveness as technological objects or tools, whether concrete or structured movements in digital technology. They help, but are not enough to teach (Lorenzato, 2006) mathematics in the educational context of traditional teaching.

Mathematics, still from this point of view, works with various contents, concepts, definitions and rules to solve mathematical problems, because the method as teachers include in the classroom is of great relevance, because the methodology that is taught, reflects on student learning. Albrecht and Maciel (2020) say that teachers need to change the way they teach, thinking about how to deal with people's everyday problems.

The didactic activity was structured based on the TSD, and Brousseau (1997) proposes the didactic triangle (Figure 1), which includes three fundamental elements - the student, the teacher and the knowledge--that are fundamental parts of an interactive and complex relationship--the didactic relationship--that takes into account the discussions between teachers and students (human elements), mediated by the mathematical knowledge (non-human element), which determines how these relationships will be realized in the educational context. The didactic relationship involves four phases: action, formulation, validation and institutionalization.

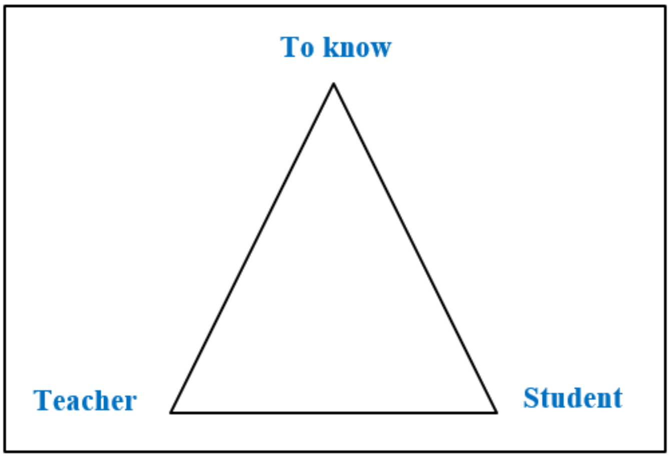

Figure 1. Didactic triangle, Brousseau (1997)

According to Brousseau (2006), the action situation is related to the construction and deconcentration of the student's knowledge processes in the classroom. In the formulation situation, participants organize themselves to share information and communicate well (Brousseau, 2006). According to Brousseau (2006), the validation situation is the exchange of assertions and solutions found by class participants to reach a conclusion about knowledge.

In institutionalization, the teacher's main function is to transform the students' discoveries in the context into "stable, decontextualized and institutionally accepted knowledge.means, such as authority and strength" (Gosztonyi, 2017, p. 1737). The didactic contract, the teacher expects the student to follow the rules and complete the proposed steps (Brousseau, 1997).

Saunders, Lewis, and Thornhill (2020) propose that the researcher teacher present new methods not only by observing, but also by presenting aspects that can be adapted, such as the advantages of including qualitative research in the situation presented during the application of the game.

The study of numerical expressions is the focus of this work, since it involves properties that can alter the mathematical thinking of students when dealing with complex problems, which, in general, are perceived in a complex way, being worked on learning the contents of the basic mathematical operations of initial basic education.

According to da Silva and de Arruda (2011), the numerical expression is an operation that uses numbers to calculate other numbers with symbols. The result is a single number, which is divided into several parts. The authors explain that, "[...] expressions (parentheses), [brackets] and {keys}, what is in the parentheses is done first, then the bracket and finally what is in the key, in the order they appear in the expression" (da Silva & de Arruda, 2011, p. 26).

EJA students have in mind several paths to be followed, questioning whether the process established by these rules is valid for all mathematical operations. But it is not enough to teach everything, it is necessary to attract all students to mathematics and technology in the classroom.

RESEARCH METHODS

The research in question followed the characteristics of applied research, with a qualitative approach and descriptive-exploratory objectives (Gil, 2008). The systematic of studies and activities aimed at the reconstruction of algebraic knowledge is present in the curricular components of Mathematics Teaching, including conceptual and experimental activities using the WordWall platform (https://wordwall.net/) for the elaboration of a gamification for EJA students.

Theoretical Assumptions in the Methodology

The research objectives are descriptive due to the description of a given phenomenon, analyzing its characteristics within a group of data; exploratory due to proportionality on the object of study (Gerhardt & Silveira, 2009). The qualitative approach considers the subjectivity and the absence of statistical analysis to measure the data collected in the research (Zanella, 2013).

Development of the Application

During the realization of this study, a bibliographic research was carried out on the theme in question, which consisted of scientific articles, abstracts of scientific events and book chapters extracted from Google Scholar and HAL Science Ouverte that addressed the application of gamified activities in basic education based on TSD.

After an analysis of some theoretical assumptions that guide the application of Gamification, a game was created using the WordWall platform. WordWall is used in classes through games, to present the disciplinary contents, using different types of games. Thus, it is feasible to create an interactive game that combines questions and images that define the topics to be addressed.

Locus and Participants of the Research

The activities were developed at the Assis Bezerra Full-Time School, located in the municipality of Quixeramobim, which operates at night with EJA Qualifica, totaling six hours of classes per week. These classes were divided into six meetings to teach about numbers, their properties and applications. Two meetings we used the WordWall platform to collect information and another four for analysis and description of the data collected.

The research was conducted with twenty-three students of the Mathematics subject in the EJA class of 2021. The activities were designed according to the curriculum plan of Youth and Adult Education, and their elaboration was based on the basic contents of numerical expressions.

Thus, the research began with an activity called Find the Expressions, which is a game with random sequences of numerical expressions that results in a positive or negative integer. The student sees the mathematical representation in the game and the results found during each question, providing the results and discussions described in the next topic of the work.

RESULTS AND DISCUSSION

The didactic contract with the EJA class, proposes a decision making in the classroom, explicitly or implicitly, about any method, conviction or knowledge of mathematics. Brousseau (1986, p. 50) defines the didactic contract as "the rule of the game and the strategy of the didactic situation".

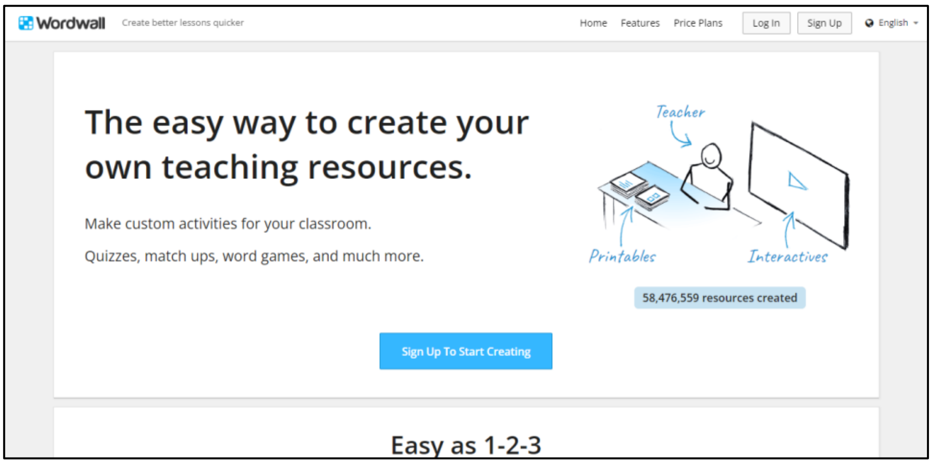

Figure 2. WordWall platform on the home page

Research Results

Digital games were used in this pedagogical practice: the storyline to encourage student involvement in the game, the objectives to direct efforts and point out what should be done during the game; Scores that allow to evaluate how many hits were per student in the cognitive dimension; Feedbacks on the importance of visualizing errors and correcting them, leading to the learning of the contents of the four operations and numerical expressions inserted in the game of the WordWall platform and; the Achievements after the completion of the activity, in the sense of assigning an evaluative note.

The application of gamification is based on the prior knowledge of EJA students, which makes possible a fundamental situation constructed as a didactic situation in the teaching of numerical expressions that promotes interaction with the WordWall platform. The stages of the TSD that are based on the research are:

The action stage is when students make decisions, including their knowledge in practice to solve the problem. It is the moment when we start learning the knowledge that is not expressed mathematically.

The activity showed the basic relationships of the four operations of mathematics. Tolomei (2017), showed how students can interact and learn from mistakes and successes during the application, and how they can receive feedbacks at the end of the game. In the Formulation stage, students create a way to explain the problem, transforming their implicit knowledge into explicit, where the subject resumes his action at a higher level and appropriates mathematical knowledge in a coherent way.

In the validation stage, the students' strategy is presented to the class. It is the moment when we start learning the knowledge that is not expressed mathematically.

Each group or team proposes a strategy to solve the problem, and may contest with other students in the same class.

Feedbacks on mistakes and successes are included in the institutionalization phase. The mathematical character of the students is proven. This is a description of what was built during the previous stages, with a meaning defined by the research participants.

The teacher has a dynamic role searching and organizing the situations that will be recorded in the classroom. It is evident that, by analyzing the responses of each student, it is feasible to improve skills related to digital fluency, independence and critical thinking (Martins, Giraffa, & Lima, 2018).

During the application of the game in the action stage, when it was possible to verify the mathematical concept of the numerical expression (Figure 3), recorded comments of the interactive moments of the students renamed by E1, E2, E3, ..., E23, were recorded. E23, in particular E6 describes the following mathematical thought "first calculate what is inside the brackets and then perform with the numbers that are outside, resulting in the correct option".

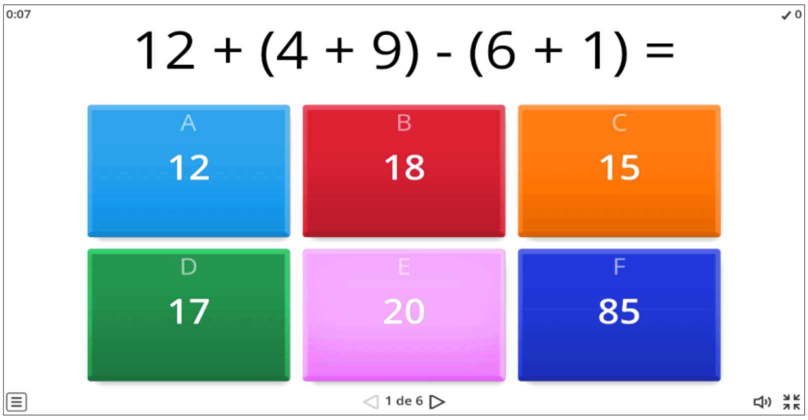

Figure 3. Gamification of numerical expressions in WordWall

Discussion

At that moment, it was possible to analyze the appearance of a level of difficulty on the part of some students, such as: eliminating the signs already used, what is the sequence of the expression and play of positive sign with negative, the following question asked to all, for example "can we use other methods to solve the question, is there an easy way to solve?".

It is noticeable that, when applying the game activity, the teacher had to assume a questioning and mediating posture in relation to the use of the platform. Valente (1997) mentions that, by moving from the level of doing to understanding computers as a pedagogical tool in the classroom, it was possible to build concepts and develop skills resulting from a change in the educational process.

By emphasizing the emphasis on the basic operations of mathematics, during the institutionalization stage, the teacher explained some resolution methods on the demonstration of numerical expressions and the four basic operations, checking each property related to the content worked during the classes.

With the students' discussions and comments on the inclusion of the gamified activity, it was realized that the game developed brought benefits, such as the motivation of the students in the activity; the reduction of time in the evaluation process in the classroom; raised the understanding of the students and increased the level of student autonomy, with a view to the calculations elaborated.

The class included other demonstrations, extending to the situation of basic content learning. This interpretation is broader, as it apparently requires students to have knowledge about the subject during their study time.

In both answers, it is clear that there is a different interpretation, where the specific content not studied by EJA students can be seen as a different thought when solving numerical expressions. When calculating the expressions, it is possible to notice that the results were determined by the numerical sequence of each question, which results in problems that can be interpreted differently by each student, who prefers to look for methods and ways to solve the mathematical properties that were previously explained by the teacher in the classroom.

Gamification joined the objectives of the lesson plan, evidenced in the results of each participant. First, the student had to "understand the calculations of the four mathematical operations", elaborate their resolutions of each question; the second, the "application of numerical expressions", define what are the orders of resolution of each sign or expression, and the third, "success in completing all the questions", if the student achieved a score between seven and ten points.

When performing the activity, it was possible to notice that the solutions of each student are satisfactory for learning mathematics, since the way they interact and follow each mathematical step of the content was intuitive and meaningful for students and educators during Didactic Transposition.

CONCLUSION

By employing mathematical concepts and numerical expressions with the aid of technologies at different levels of knowledge, it is possible to affirm that the use of digital technologies for educational purposes offers possibilities of satisfactory results for EJA students, in addition to allowing an innovative approach to learning with new digital tools.

Thus, the objective of the work was achieved by applying the contents with interactive methods by the teacher, providing collaborative teaching with the use of technology to improve the teaching and learning process of mathematics. Therefore, it is crucial that the educational institution is an environment in which knowledge is intuitive and interactive for students, being a space for the exchange of knowledge and applications that can be worked with everyday experiences.

The work demonstrated that the methods used by the teacher in the classroom to teach and learn mathematics with EJA students reinforced pedagogical actions structured in the contribution of fundamental content.

It also increases students' interest, motivation and participation. It is important to note that a participatory and intuitive spirit is developed from the hypotheses presented by the students during the resolution of the problem, based on the Theory of Didactic Situations.

It also increases participation in the practice described, facilitating the problematization of the contents addressed in this work with the teaching of mathematics, in order to compare the mathematical contents to their reality to interact and encourage learning in the classroom, contributing to the formation of mathematics.

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Effectiveness of Problem Based Learning Models Assisted by Worksheets on Students' Critical Thinking Ability

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PBL Model;
Worksheet.

Abstract

This study aims to describe the effectiveness of the Problem Based Learning (PBL) model assisted by worksheet on the critical thinking ability of students. This quasi-experimental study employs the nonequivalent posttest-only control group design with the cluster random sampling technique. The study sample consists of students from classes VII B and VII C in one of the Public Junior High Schools in Magelang, Central Java, Indonesia. The research instruments used are learning tools in the form of worksheet and critical thinking ability tests. The results of this research are; 1) students who are taught using the PBL learning model assisted by worksheet against critical thinking ability are said to be effective by obtaining learning completion has reached 75%, 2) students who are taught using the direct learning model against critical thinking ability are said to be ineffective by obtaining learning completion has not reached 75%, 3) critical thinking ability of learners taught using the worksheet-assisted PBL model more effective than using the direct learning model. This study implies differences in students' critical thinking abilities when comparing the worksheet-assisted PBL learning model to the direct learning model. Education needs to apply appropriate learning models to develop students' critical thinking skills.

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INTRODUCTION

One of the abilities that must be possessed by students is the ability to think critically. Of course, a teacher must play an important role in the process of developing students' critical thinking and logical thinking skills. This is in line with Lismaya (2019) explaining that critical thinking is an intellectual process by starting the step by creating a concept, applying, and doing synthetic or evaluating the knowledge that has been obtained through observation, experience, reflection, basic thinking to perform an action. Furthermore Surya (2011) explains that critical thinking is an active, persistent activity and carefully considers a decision or form of knowledge received, seen from various angles of reasons that support and conclude. The ability to think critically is essential in delving into new material and relating to what learners already know, even if learners cannot know it all. Learners can learn to ask questions effectively and reach conclusions consistent with the facts.

According to Ennis (2011), critical thinking skills are grouped in the top five activities including (1) Elementary clarification, explaining simply by focusing the question on the problem, (2) Basic support, conducting observation and considering and observing the credibility of the source of completion, (3) Inference, making and considering of deductions, induction, and decisions, (4) Advanced clarification, making further explanations, namely defining terms and considering definitions, and (5) Strategy and tactics, managing strategies and tactics by performing an appropriate action in problem solving. Furthermore, Facione (2011), the ability to think critically has six basic skills, including (1) Interpretation, understanding a problem, (2) Analysis, the process of analyzing the relationships between concepts and statements, (3) Evaluation, determining and using the right strategy in solving problems, (4) Inference, making a conclusion, (5) Explanation, providing a reason based on the conclusions drawn and (6) Self-regulation, looking back at the answers given based on the ability of oneself to see the achievement of the expected goals. Critical thinking skills applied include (1) Focus, focusing on problems by providing problems with problems, (2) Analysis, identifying relationships between concepts and statements, (3) Evaluation, using appropriate problem-solving strategies, and (4) Inference, providing conclusions appropriately.

Based on discussions with grade VII mathematics teachers at one of the Public Junior High Schools in Magelang, Central Java, Indonesia, students are less active in the learning process so that learning tends to be passive. Students have not been able to express what is not yet understood and have not dared to explain what the teacher asked. Observations at there show that the learning process applied is teacher-centered. The results of the initial test of critical thinking ability of grade VII students showed that the average score obtained was 16.15 out of a maximum score of 100. These results are categorized very low based on the category of critical thinking ability according to Arikunto (2018). This shows that a teacher must be able to create a learning atmosphere that can train students' critical thinking skills with an appropriate learning model.

One alternative learning model that can train students' critical thinking skills is to apply PBL learning model. PBL is one of the learning models designed to develop students' problem-solving abilities. According to Jailani, Sugiman, and Apino (2017), PBL is a problem-based learning model. Using learning that starts from problems, students can learn concepts and principles while solving these problems. According to Octavia (2020), PBL is a learning that presents a variety of authentic and meaningful problem situations to students that serves as a foundation for investigation, learner investigation skills, and problem-solving skills. This is in line with Isrok'atun and Amelia's explanation (2018) that PBL is a learning approach that begins by exposing students to a real-world problem and guiding them to solve it through experiences gained during the learning process. The PBL model is thus a learning that focuses on problem-solving activities in real life, thus training students to use thinking skills in developing ideas to solve a problem skillfully, play an active role in learning, and confidence in making decisions.

PBL research that has been conducted by Setyaningsih and Abadi (2018) shows that mathematics learning with PBL with a collaborative group and collaborative setting in pairs is effective in terms of algebraic learning achievement, critical thinking ability, and student anxiety. PBL with a group collaborative setting is better than a paired collaborative setting in terms of algebraic learning achievement, but there is no difference in effectiveness. The results of another study conducted by Yuni, Bharata, and Caswita (2017) found that there was an influence of the PBL model on the mathematical critical thinking ability of students.

One of the learning media that can be applied to the PBL model is the Student Worksheet. This is strengthened by the results of research by Azmi, Rahayu, and Hikmawati (2016) that the PBL model with the help of worksheet can provide opportunities for all students to play an active role in solving problems both by means of experiments and group discussions. Worksheet is one of the alternative learning resources that can be applied because it can help students to add information about the material learned through structured learning activities (Islamiah, Rahayu, & Verawati, 2018). Worksheet usually contains titles, instructions for use, basic competencies, supporting information, tasks and steps to complete a task and assessment of learning objectives.

RESEARCH METHODS

This research is a type of quantitative research using quasi-experimental methods. The design model used is the nonequivalent posttest-only control group. In this study, there were free variables, namely the PBL learning model assisted by worksheet (X) and the direct learning model (Y), while the bound variables used were critical thinking skills. There are two classes in this study, namely the group given the worksheet-assisted PBL model treatment (X) called the experimental class (P_1) and the group given the direct learning model treatment (Y) called the control class (P_2). After being given treatment, a posttest was then carried out to measure critical thinking skills in both classes. The experimental class was given posttest questions (O_1) and the control class was given posttest questions (O_2). The research design can be seen in Table 1.

Table 1. Quasi experimental research design

Research class	Treatment	Posttest
P_1	X	O_1
P_2	Y	O_2

This research was conducted at one of the public junior high schools in Magelang, Central Java, Indonesia in the 2022/2023 academic year. The population in this study was all class VII students with a total number of 221 students. This study used class VII B as an experimental class and class VII C as a control class. The number of students in each class is 32 students, so the number of samples is 72 students. Sampling was carried out using the cluster random sampling technique.

The instruments used in this study were learning tools in the form of worksheet and critical thinking ability tests. Worksheet is used to assist the process of delivering learning materials by applying the learning steps of the PBL model. Meanwhile, the test instruments used are first validated and tested. Test instrument validation consists of content validation and construct validation. Validation of the contents of the test instrument is based on the results of the assessment by both validators with three statements declared worthy of testing. Meanwhile, construct validation is carried out after testing the test instrument. Summary of the results of the experimental item analysis of the critical thinking test instrument in Table 2.

Table 2. Summary of instrument calibration analysis results

No.	Contents validity	Construct validity	Reliability	Difficulty level	Difference power	Conclusion
1	Highly	Valid	Very	Keep	Good	Used
2	Valid	Valid	Good	Keep	Bad	Discarded
3		Valid		Difficult	Bad	Discarded
4		Valid		Difficult	Bad	Discarded
5		Valid		Keep	Good	Used
6		Valid		Keep	Good	Used
7		Valid		Difficult	Good	Used
8		Valid		Difficult	Good	Used
9		Valid		Difficult	Enough	Discarded
10		Valid		Difficult	Enough	Discarded

Based on Table 2 of the questions used for the posttest, there are five question items. Three points of question at a moderate level of difficulty with good differentiability. Two points of question at the difficulty level are difficult with sufficient differentiability. The collection techniques carried out in this study were observation, interviews, tests and questionnaires as well as documentation. Test instruments are used to determine learners' critical thinking skills, while questionnaire instruments are used to validate test instruments.

The data analysis technique used in this study is the *Z* statistical test to determine the completeness of learning model, while the *t*-test is used if the data is normally distributed and homogeneous to determine the average difference between the two samples. Meanwhile, the Mann Whitney test if the data obtained is not normally distributed or inhomogeneous to find out the difference between the two samples from the posttest value results. However, before conducting the test, a prerequisite test is carried out consisting of a normality test and a homogeneity test.

RESULTS AND DISCUSSION

The effectiveness of the PBL model assisted by worksheets on students' critical thinking is evaluated through two stages: the achievement and ability difference tests. Prior to these, a prerequisite test is conducted to identify the analytical tools for the assessments, one of its benefits.

Prerequisite Test

The prerequisite tests carried out include normality and homogeneity tests. Normality testing using the Shapiro wilk sample test and homogeneity testing using the homogeneity of variances test. Normality tests are carried out to determine whether the experimental class and control class are normal or not. Summary of the normality test results presented in Table 3.

Table 3. Summary of normality test results

Research class	*df*	*p*-value	Conclusion
P_1	26	0,000	Non-normal distribution
P_2	24	0,000	Non-normal distribution

Based on Table 3, it was obtained that after posttesting in both classes, a significance value of 0,000 was obtained. The value is less than the allowed significance value (p-value=0,000<0,05=α), which means that the results of the posttest normality test in both classes are not normally distributed.

Furthermore, a homogeneity test is carried out. Homogeneity test is carried out to determine whether both classes are homogeneous or not. Summary of the homogeneity test results is presented in Table 4.

Table 4. Summary of Homogeneity Test Results

	Levene statistic	*df*	*p*-value	Conclusion
Mean	1.618	48	0.210	Homogene
Median	0,418	48	0,524	

Based on the calculation results of the Homogeneity test it was obtained that after posttesting in both classes, a significance value of 0.210. The value is greater than the permissible significance value (p-value=0.210>0.05=α), which means that the two classes are homogene.

Based on the results of the normality and homogeneity test of the posttest of critical thinking ability, data were obtained not normally distributed and homogene. The data obtained were not normally distributed, followed by the Mann Whitney test to determine the difference between the experimental class and the control class.

Achievement Test

The results of the analysis of the test show the percentage of student learning completion in the PBL model assisted by worksheet and direct learning model, presented in Table 5.

Table 5. Result learning completion of the PBL model

Class	*N*	Mean	*Z*	$Z_{\alpha=0.05}$	Conclusion
P_1	26	74	0.68	1.64	Completeness score has reached 75%
P_2	24	65,6	-1,89	1,64	Completeness score has not reached 75%

The test criteria used are if $Z > -Z_{\alpha=0.05}$, so students' completeness score reached 75%. Based on the calculation results obtained: a) On students' group within the PBL model (P_1), Z=0.68, and the value is greater than $-Z_{\alpha=0.05}$, it means that learning completion testing of students taught using the worksheet-assisted PBL model achieved learning completion by obtaining a score of more than 70 and reached 75%; b) On students' group within the direct learning model (P_2), Z= –1.89, and the value is less than $-Z_{\alpha=0.05}$, it means that the completeness of learning for students who are taught using a direct learning model achieves learning completion by obtaining a score of more than 70 but has not reached 75%.

Ability Difference Test

The results of the analysis of Mann Whitney test, to determine whether there is a significant difference in critical thinking abilities between students group within worksheet-assisted PBL model (P1) and direct learning model (P2), are presented in Table 7.

Table 7. Mann Whitney posttest test results

Class	*N*	Mean	*U*	*Z*	Conclusion
P_1	26	74	124	–3,65	Students' critical thinking differed significantly
P_2	24	65,6	499		

Statistical value of *U* first converted into the form of a standardized normal *Z* value (Suyanto & Gio, 2017). The test criteria used are if *Z* is greater than $Z_{\alpha=0.05}$, so the students' critical thinking differed not significantly. Based on the calculation results obtained $Z = –3,65$ in the table the cumulative normal distribution is 0.0002 with two-way testing, so the students' critical thinking differed significantly. The Mann Whitney test obtained results that there was a difference between the critical thinking skills of the experimental class (P_1) and the control class (P_2). After that, judging from the average posttest score obtained, the critical thinking skills of experimental classes taught using the worksheet-assisted PBL model were more effective than the critical thinking skills of control classes taught using direct learning models.

Students' Critical Thinking by Worksheet-Assisted PBL Model

Sourced from the learning completeness test of the PBL learning model obtained $Z = 0.68 > –1.64 = Z_{\alpha=0.05}$. Result of *Z* is located in the reception area, so that the percentage of learning completion of students' critical thinking ability taught using the worksheet-assisted PBL model has reached 75%.

In the experimental class (the worksheet-assisted PBL model), posttest was attended by 26 students with 21 completed students, while 5 students who had not reached completion in learning. It can be said that in the experimental class taught using the worksheet-assisted PBL model, students' critical thinking skills have reached learning completion of 80.8% which can be categorized in the high category. Supported by the results of research from Azizah, Sugiyanti, and Happy (2019), it was obtained that the mathematical critical thinking ability of students using the PBL learning model has achieved classical learning completion of 92.28% with 26 completed students from 28.

In the first stage, presenting a problem students are divided into several groups to understand the given problem contained in worksheet. At this stage, train students to focus on the problems given by the teacher. The first meeting was given value comparison material and the second meeting was given value reversal comparison material, where students were asked to understand the concept of the material given.

In the second stage, exchanging information tries to find the right step in solving the problem by exchanging opinions within each group. At this stage, train students to analyze the relationship of questions, statements, and concepts to the given problem. At the first meeting, students were still hesitant to present arguments to their groups during the discussion activity. In the second meeting, students have begun to be more active than previous learning and have dared to present the arguments needed to each other in the discussion activity.

In the third stage, solving problems outside the guidance of the teacher, where learners are given the freedom of how to solve problems with relevant sources. At this stage, train and give freedom to students to find and determine the right problem-solving strategies. At the first meeting there were some students who did not cooperate in their groups on problem-solving activities. In the second meeting there were changes during the learning process, where students became more active and very enthusiastic in working together to find solutions to the problems given.

In the fourth stage, presenting solutions, where students write down and present the solutions to the problems made to other groups. In the first meeting, some students divided group tasks well and there were some groups that divided tasks unevenly in presenting solutions. In addition, in presenting the results of group discussions, students hesitate to convey the results of discussions and group work to other groups. In the second meeting, where students become better than previous learning. This is because students become more enthusiastic and enthusiastic in working together to solve problems given with a good distribution of tasks. In addition, in presenting the results of group discussions and group work to other groups, there are more than two groups that deliver in front of the class.

In the fifth stage, reflecting, namely students expressing opinions related to things or material obtained during learning. In the first meeting, there was only one student who dared to convey the results of the material that had been learned, while for the second meeting, there were several students who dared to convey what had been learned during learning.

The PBL learning model provides an opportunity for learners to become more active in the learning process. This is shown in the learning step in the second stage to the fifth stage. In addition, the PBL learning model applied can help develop students' critical thinking skills. In line with Swiyadnya, Wibawa, and Sudiandika (2021), the PBL model provides real conditions for students by directing authentic learning and providing new information to students and making students continuously seek and find solutions to problems that can be utilized in life. This can be said through activities to exchange opinions or ideas and solve problems presented with the help of worksheet.

Students' Critical Thinking by Direct Learning Model

Sourced from testing the completeness of learning models directly obtained $Z = -1.89 > -1.64 = Z_{\alpha=0.05}$. Result of Z located outside the reception area, so that the percentage of learning completion of students' critical thinking ability taught using the direct learning model has not reached 75%.

In the posttest control class (the direct learning model), there were 24 students with 14 students who were complete in learning, while 10 students who had not reached completion in learning. It can be said that in the control class taught using a direct learning model on the critical thinking ability of students has not reached learning completion of 58.3% which can be categorized in the category of enough. Supported by the results of research from Azizah et al. (2019), it was obtained that the mathematical critical thinking ability of students using the direct learning model has not achieved classical learning completion of 53.57% with 15 completed students out of 28.

In the direct learning model, the instruction is teacher-centered in both the first and second meetings, so that students are less active and hesitant when they want to ask questions about learning materials or answer questions from the teacher. During learning, the teacher tries to build interaction in learning so that students become active and prevent students from becoming bored during learning and prevent students from becoming engrossed in their own activities. Direct learning activities do not provide opportunities for students to convey ideas or ideas in view of the learning steps. Learners obtain material based on submissions made by teachers without their own efforts to obtain material.

Difference in Students' Critical Thinking Abilities

Referring to Siregar (2015), based on the posttest average obtained, experimental classes taught using the worksheet-assisted PBL model were more effective than control classes taught using direct learning models on critical thinking skills. This result is in line with research from Mulyanto, Gunarhadi, and Indriayu (2018), that there are differences in learning outcomes seen from the ability to think critically using a PBL model with a direct learning model. The results of this study are also supported by the research of Prihono and Khasanah (2020), it was found that PBL learning has an influence on students' critical thinking skills, as well as critical thinking skills taught using the PBL learning model better than using the direct learning model.

Based on the averages obtained, experimental classes taught using the PBL model are better or more effective than control classes taught using a direct learning model of critical thinking ability. These results are in line with research from (Mulyanto, Gunarhadi, & Indriayu, 2018), that there are differences in learning outcomes seen from the ability to think critically using a PBL model with a direct learning model. The results of this study are also supported by a study conducted by Prihono and Khasanah (2020), the results were obtained that PBL learning has an influence on students' critical thinking skills, as well as critical thinking skills taught using the PBL learning model better than using the direct learning model.

The relationship between the PBL model and worksheet learning media lies in the content of worksheet which can contain the steps of the PBL model and indicators of critical thinking skills. The PBL learning model using worksheet media plays an important role in the success of the learning process. Worksheet is very helpful for students to achieve the learning goals to be achieved. The activities contained in worksheet have been adjusted to the indicators of students' critical thinking skills and in accordance with the material studied. The steps contained in worksheet direct students in developing ideas for critical thinking skills. Students seem very enthusiastic in carrying out the activities requested in worksheet. This is in line with Florensia, Yurnetti, and Hamdi (2018), that worksheet in which there is a PBL model step becomes an exercise in developing learning aspects by changing one-way teaching habits in dominating teacher activities aimed at training and developing students' thinking skills, so that students can be more active in the learning process.

The successful application of the worksheet-assisted PBL model to the thinking ability of students found during the implementation of learning is 1) worksheet-assisted makes students able to understand the material more easily and develop the ability to solve problems, 2) increase student learning motivation which makes students more enthusiastic in the learning process, and 3) provide opportunities for students to develop innovative ideas or ideas in their critical thinking ability to understand problems in real life.

The PBL learning model using worksheet media plays an important role in the success of the learning process. Worksheet is very helpful for students to achieve the learning goals to be achieved. The activities contained in worksheet have been adjusted to the indicators of students' critical thinking ability and in accordance with the material studied. The steps contained in worksheet direct students in developing ideas for critical thinking skills. Students seem very excited in carrying out the activities requested in worksheet. The application of worksheet in PBL learning stimulates the activeness of students, so that students are more challenged to determine the right steps in solving the problems found. The application of worksheet received a good response from students because students looked more enthusiastic in learning.

The learning process carried out in the control classroom is taught using a direct learning model, students tend to be more passive, so it feels boring and the learning applied is teacher-centered or learning that takes place in one direction. In line with the opinions of Rachmawati and Rukmi (2014), explained that an obstacle that exists in the direct learning model is that the classroom atmosphere during learning becomes rowdy. This can be one of the obstacles in the process of developing students' critical thinking skills.

The teacher provides an explanation related to the learning material accompanied by sample questions and steps to solve them directly. Practice questions are given after an explanation of the material from the teacher to be done by students, where practice questions are done with problems that are often encountered. Students do practice questions given in groups assisted by guidance from the teacher. Some students do practice questions according to the direction of the teacher, but there are still those who do practice questions by writing answers without any steps.

During learning in the control class, the teacher tries to implement more interactive learning by appointing several students to deliver their work in front of the class. This does not get a positive response from students, thus making learning less effective and making students less to explore themselves better.

CONCLUSION

Based on the results and discussion above, it can be concluded that 1) students who are taught using the PBL learning model assisted by worksheet against critical thinking ability are said to be effective by obtaining learning completion has reached 75%, 2) students who are taught using the direct learning model against critical thinking ability are said to be ineffective by obtaining learning completion has not reached 75%, 3) critical thinking ability of learners taught using the worksheet-assisted PBL model more effective than using the direct learning model.

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The Influence of Mathematical Logical Intelligence on Student Learning Outcomes in Linear Algebra Courses

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Abstract

The purpose of this study was to determine the effect of mathematical logical intelligence on student learning outcomes in Linear Algebra courses. The research method used is quantitative. The sampling technique used in this study was simple random sampling. Data collection in this study used a mathematical logical intelligence test of 20 multiple choice questions and a test of student learning outcomes in linear algebra courses of 5 description questions. The variables in this study consisted of independent variables and dependent variables. The independent variable in this study is mathematical logical intelligence (X), while the dependent variable in this study is student learning outcomes in Linear Algebra courses (Y). The instrument test in this study consisted of validity test, distinguishing power, difficulty level and reliability test. The classical assumption test in this study consists of normality test and linearity test. The data analysis technique used simple linear regression. The result of this study is that there is an influence between mathematical logical intelligence on student learning outcomes in linear algebra courses of 0.527 or 53%. While the other 47% is influenced by other variables outside this study.

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INTRODUCTION

Linear Algebra is one of the basic compulsory courses in the Informatics Engineering study program taken in semester 2. The material discussed in the Linear Algebra course is matrices, systems of linear equations, vectors, eigenvalues and eigenvectors. Since linear algebra is a compulsory course, it is expected that students can master the course well. However, in reality there are still many students whose mastery of linear algebra material is still low.

The low mastery of linear algebra material is because some students still consider linear algebra to be a difficult course so that students' desire to try to practice linear algebra material is quite low. Tristanti and Nusantara (2023) stated that one of the subjects that is difficult to learn for students is linear algebra.

Students tend to only write back and imitate what has been explained by the teacher. If given a problem that is slightly different from the example, students tend to have difficulty. So, this causes student learning outcomes also tend to be lacking. However, there are some students who tend to actively ask if there is material that they do not understand, the desire to try to solve problems is also good, so that these students get good learning outcomes as well. Learning outcomes are the results or results that a person gets after going through the learning process (Lestari, 2014). Learning outcomes are also the final results obtained by students after participating in the learning process which is marked by a value scale in the form of letters or numbers that can be used as a measure of whether or not students are successful in learning mathematics (Firmansyah, 2015). Based on several opinions regarding learning outcomes, it can be concluded that linear algebra learning outcomes are the final results in the form of grades obtained by students after following the Linear Algebra lecture process.

Low student interest and motivation in courses related to numbers can affect mathematical logical intelligence. The amount of material related to variables in linear algebra courses makes students less interested in the material. In addition, the low ability of students to solve exercise problems that are different from the examples given, shows that the logic of students does not run well or does not run as it should. The material in the Linear Algebra course is mostly related to numbers, calculations, variables ranging from matrices to eigenvectors. According to Sudrajat, the lack of student understanding causes linear algebra to be considered an abstract, complicated and boring subject, causing students to be unmotivated in learning linear algebra (Dafid, Wibawa, & Situmorang, 2022). If student interest is low in linear algebra courses, it can affect the acquisition of student learning outcomes.

There are 2 factors that influence student learning outcomes in linear algebra, namely internal factors and external factors. External factors include environment, family, friends. Meanwhile, one of the internal factors is the intelligence that each person has. There are 8 kinds of intelligence categories according to Gradner, namely linguistic intelligence, mathematical logical intelligence, visual and spatial intelligence, musical intelligence, interpersonal intelligence, intrapersonal intelligence, kinesthetic intelligence and naturalist intelligence (Shahrokhi, Ketabi, & Dehnoo, 2013). Mathematical logical intelligence is closely related to mathematics (Supardi, 2014). Mathematical logical intelligence is the ability to use numbers effectively and reason well (Al-Salameh, 2012). Logical thinking skills are needed by individuals during activities in making decisions, drawing conclusions and solving problems, where someone who has mathematical logical intelligence has the characteristics of being able to think according to logical rules, based on structure, according to appropriate rules, classifying, categorizing and being able to analyze numbers (Sari, 2020). Nurzaelani and Wibowo (2015) stated that mathematical logical intelligence is very important for every student, especially in mathematics, where the material includes algebra, measurement and geometry, opportunities, statistics, trigonometry and calculus. From several opinions that have been described, mathematical logical intelligence is the ability to count using numbers, the ability to think logically and the ability to solve mathematical problems that use variables.

The characteristics of people who have prominent logical intelligence include having a strong ability to reason, sequence, think in causal patterns, create hypotheses and look for conceptual regularities or numerical patterns (Ula, 2013; Nurzaelani & Wibowo, 2015). The characteristics of mathematical logical intelligence are having prominent abilities in the fields of logic, causal patterns, classification and categorization, abstraction, symbolization, inductive and deductive thinking, counting, playing with numbers, scientific thinking and syllogism (Widiastuti, 2015). Based on several opinions regarding the characteristics of people who have mathematical logical intelligence, there are 4 indicators of mathematical logical intelligence used in this study, namely students are able to solve problems related to calculation (numerical), students are able to solve problems related to algebra (containing variables), students are able to solve problems about patterns / series, students are able to solve problems related to logic.

In the research conducted by Dafid et al. (2022) provides results that mathematical logical intelligence makes a major contribution to improving student learning outcomes in Linear Algebra lectures. Sari (2019) in the research conducted obtained the results that there is a significant influence between mathematical logical intelligence and student learning outcomes, which means that students with high mathematical logical intelligence will be easy to learn and understand mathematics so that students are quick in solving math problems. While research by Santoso and Utomo (2020) obtained the result that mathematical logical intelligence partially has no significant effect on student learning outcomes.

Some research results show that mathematical logical intelligence does have an influence on learning outcomes, but there are also some who say that mathematical logical intelligence has no effect on learning outcomes. Therefore, this study tries to determine the magnitude of the influence of mathematical logical intelligence on the learning outcomes of undergraduate students of Informatics Engineering in Linear Algebra courses. In connection with this description, the purpose of this study is to determine the magnitude of the influence of mathematical logical intelligence on student learning outcomes in Linear Algebra courses.

RESEARCH METHODS

The research conducted included a type of quantitative research. The subjects of this study were 1st year students in the 2022 Linear Algebra course, namely classes 1D, 1E, 1F and 1J. The population used was 130 students. The sampling technique used in this study was simple random sampling. According to (Riduwan, 2003) simple random sampling is a way of taking samples from members of the population at random without paying attention to the strata / levels in the population members. Determination of the number of samples using the Slovin formula (Putri & Kartika, 2017; Wahyuniar et al., 2021). From the calculation using the formula, the sample used in this study was 56 students. Because the number of students in each class is not the same, the sampling from each class is 18 students taken from classes 1D and 1E, 13 students from class 1F and 7 students from class 1J.

The independent variable (X) used in this study is mathematical logical intelligence, while the dependent variable (Y) is student learning outcomes in Linear Algebra courses. Data collection techniques for mathematical logical intelligence and linear algebra learning outcomes are by using tests. For the mathematical logical intelligence test in the form of multiple choice. There are 4 indicators used for the mathematical logical intelligence test which are presented in Table 1.

Table 1. Logical mathematical intelligence test indicators

Indicator	Number of items
1. Students are able to solve problems related to calculation (numerical)	5
2. Students are able to solve problems related to algebra (containing variables),	5
3. Students are able to solve problems about patterns/sequences	5
4. Students are able to solve problems related to logic	5

Table 1 is an indicator of the mathematical logical intelligence test, where there are 4 indicators for the mathematical logical intelligence test. Each indicator consists of 5 questions/question items. Thus, there are 20 questions for the mathematical logical intelligence test.

As for the linear algebra learning outcomes test in the form of descriptions. For the student learning outcomes test in linear algebra courses there are 5 indicators presented in Table 2.

Table 2. Student learning outcomes test indicators on linear algebra

Indicator	Number of items
1. Students are able to calculate operations on matrices	1
2. Students are able to solve SPL with Gauss Jordan elimination	1
3. Students are able to determine orthogonal projections on vectors	1
4. Students are able to calculate eigenvalues	1
5. Students are able to calculate eigenvectors	1

Table 2 states the indicators of the student learning outcomes test on linear algebra which consists of 5 indicators, each indicator has 1 description test question. Thus, there are 5 description questions for the student learning outcomes test in linear algebra courses.

The instrument test in this study used validity, distinguishing power, difficulty level and reliability tests which were tested on 40 students. Instrument tests consisting of validity, distinguishing power, difficulty level and reliability were carried out manually using Excel.

The classic assumption tests used are normality test and linearity test. Then the data analysis technique in this study is simple linear regression. The classic assumption test and data analysis tools in this study used SPSS version 23.

The assessment of mathematical logical intelligence test refers to the formula found in Romika and Amalia (2014) and Indaswari et al. (2021).

RESULTS AND DISCUSSION

Validity Test

Nurzaelani and Wibowo (2015) stated that if r is greater than $r_{\alpha=0.05}$, then the item is considered valid, whereas if r is smaller or equal to $r_{\alpha=0.05}$, then the item is considered invalid. In this study there were 40 respondents to take the test of mathematical logical intelligence and linear algebra learning outcomes. The criteria used to determine the validity of the items is by comparing the correlation coefficient (r) and $r_{\alpha=0.05}$. The value of the $r_{\alpha=0.05}$ with N=40 is 0.312. The validity of mathematical logical intelligence items consisting of 20 questions is shown in Table 3.

Table 3. Validity of mathematical logical intelligence test

Item	*r*	Criteria	Item	*r*	Criteria
1	0,371	Valid	11	0,412	Valid
2	0,357	Valid	12	0,364	Valid
3	0,355	Valid	13	0,350	Valid
4	0,580	Valid	14	0,773	Valid
5	0,696	Valid	15	0,700	Valid
6	0,464	Valid	16	0,352	Valid
7	0,517	Valid	17	0,379	Valid
8	0,755	Valid	18	0,494	Valid
9	0,572	Valid	19	0,582	Valid
10	0,700	Valid	20	0,320	Valid

Based on the results of the calculations shown in Table 3 about the validity of mathematical logical intelligence items, it can be seen that 20 items have an r is greater than $r_{\alpha=0.05}$=0.312, so all test questions are declared valid.

Similar to the validity of items on mathematical logical intelligence, the validity of items on student learning outcomes is considered valid if r is greater than $r_{\alpha=0.05}$. The validity of the items of student learning outcomes consisting of 5 questions is presented in Table 4.

Table 4. Item validity of student learning outcomes

Item	1	2	3	4	5
r	0,68	0,82	0,90	0,87	0,89
Criteria	Valid	Valid	Valid	Valid	Valid

Based on the results of the calculations in Table 4 regarding the validity of the items of student learning outcomes, it can be seen that 5 items have an r is greater than $r_{\alpha=0.05}$, then all test questions are declared valid.

Distinguishing Power Test

A question item is said to have good differentiation if the differentiation index is equal to or more than 0.30 ($D \geq 0.30$) (Budiyono, 2011). The mathematical logical intelligence discriminating power (DP) test is presented in Table 5.

Table 5. Differentiating power of mathematical logical intelligence

Item	*DP*	Criteria	Item	*DP*	Criteria
1	0,35	Good	11	0,35	Good
2	0,35	Good	12	0,40	Good
3	0,35	Good	13	0,35	Good
4	0,60	Good	14	0,70	Good
5	0,60	Good	15	0,60	Good
6	0,40	Good	16	0,30	Good
7	0,40	Good	17	0,30	Good
8	0,65	Good	18	0,40	Good
9	0,40	Good	19	0,45	Good
10	0,60	Good	20	0,30	Good

Based on the results presented in Table 5, it can be concluded that all items on the mathematical logical intelligence test have good differentiating power because the value of differentiating power is greater than or equal to 0.30. Furthermore, the differentiating power of student learning outcomes is presented in Table 6.

Table 6. Distinguishing power of student learning outcomes

Item	1	2	3	4	5
DP	0,33	0,3	0,3	0,3	0,3
Criteria	Good	Good	Good	Good	Good

Based on the results presented in Table 6, it can be concluded that all items on the student learning outcomes test have good differentiating power because the value of differentiating power is greater than or equal to 0.30.

Difficulty Level

The level of difficulty test is carried out to determine the level of difficulty in each question including in the easy, medium or difficult category. As stated by Nurzaelani and Wibowo (2015) that the criteria for the level of difficulty of test items (*DL*) are *DL* = 0.00 – 0.30 difficult; *DL* = 0.31 – 0.70 moderate; *DL* = 0.71 – 1.00 easy. The test of the level of difficulty for mathematical logical intelligence questions is presented in Table 7.

Table 7. Level of difficulty of mathematical logical intelligence test

Item	*DL*	Criteria	Item	*DL*	Criteria
1	0,73	Easy	11	0,8	Easy
2	0,7	Medium	12	0,7	Medium
3	0,8	Easy	13	0,5	Medium
4	0,6	Medium	14	0,5	Medium
5	0,6	Medium	15	0,5	Medium
6	0,6	Medium	16	0,7	Medium
7	0,6	Medium	17	0,5	Medium
8	0,5	Medium	18	0,7	Medium
9	0,8	Easy	19	0,4	Medium
10	0,6	Medium	20	0,7	Medium

Based on the calculation results that have been presented in Table 7 regarding the level of difficulty in mathematical logical intelligence questions, there are 16 questions that have moderate criteria while the other 4 questions have easy criteria. Furthermore, the test of the level of difficulty on the question of student learning outcomes is presented in Table 8.

Table 8. Level of difficulty of student learning outcomes test

Item	1	2	3	4	5
DL	0,645	0,73	0,7	0,81	0,61
Criteria	Medium	Easy	Medium	Easy	Medium

Based on the calculation results in Table 8 about the level of difficulty of student learning outcomes in linear algebra courses, there are 3 questions that have moderate criteria while the other 2 have easy criteria.

Reliability

A variable is said to be reliable if it provides a value of Cronbach Alpha is greater than 0,7 (Riyadi & Mulyapradana, 2017). The result of the reliability test of mathematical logical intelligence is 0.85. While the reliability test result of Linear Algebra learning outcomes is 0.88. The reliability test results of the two variables show the results is greater than 0.7, so that the two variables are declared reliable and suitable for simple regression tests.

Normality Test

The normality test is carried out to show that the sample data comes from a normally distributed population. A data is said to be normally distributed if the *p*-value is greater than α=0.05 (Ghozali, 2006). The results of the normality test of the independent variable (mathematical logical intelligence) and the dependent variable (student learning outcomes) are presented in Table 9.

Table 9. Normality test results

Variable	Test-Statistic	*p*-value	Description
Logical Mathematical Intelligence	0.073	0.200>0.05	Normal
Student Learning Outcomes	0.115	0.061>0.05	Normal

In Table 9 of the normality test results, the results obtained on mathematical logical intelligence *p*-value=0.200 is greater than α=0.05, it can be concluded that the data on the mathematical logical intelligence variable is normally distributed. In student learning outcomes in linear algebra, the results of *p*-value=0.061 is greater than α=0.05, it can be concluded that the data on the variable student learning outcomes in linear algebra is normally distributed.

Linearity Test

Linearity test is conducted to determine the linearity of the relationship between the independent variable and the dependent variable. In this study, the independent variable used is mathematical logical intelligence while the dependent variable is student learning outcomes. The relationship between the independent variable and the dependent variable is said to be linear if the *p*-value on deviation from Linearity is greater than α=0.05 (Ghozali, 2006). The results of the linearity test between the two variables are presented in Table 10.

Table 10. Linearity test results

Variable	Deviation from Linearity	*p*-value
Math. logical intelligence (X) with Learning Outcomes (Y)	0.759	0.696>0.05

In Table 10, about the results of the linear test, the results show that the relationship between mathematical logical intelligence and student learning outcomes is *p*-value=0.696 is greater than α=0.05, it can be concluded that the relationship between variables X and Y is linear. After the classical assumption test has been carried out and meets all the criteria, the next step is to look for a simple regression test using SPSS.

Simple Regression Test

The results analysis of the simple regression test, is to determine the significance of the influence of mathematical logical intelligence on learning outcome, are presented in Table 11.

Table 11. Simple linear regression

Variable	Regression Coefficient	t	*p*-value
Constant	12.396		
Math. logical intelligence		7.752	0.000
F	60.088		0.000
R	0.726		
R^2	0.527		

Table 11 shows that the regression equation obtained is as follows Y=12.396+7.752X. The constant value obtained is 12.396, indicating that mathematical logical intelligence (X) if it does not change, then the amount of student learning outcomes (Y) is 12.396. The regression coefficient of the mathematical logical intelligence variable is 7.752, meaning that if there is an increase in the mathematical logical intelligence variable by one unit, it will cause the student learning outcomes variable to increase by 7.752 units. Furthermore, it is also known that F=60.088 with *p*-value=0.000 is less than α=0.05, indicating that the influence of mathematical logical intelligence on student learning outcomes is significant.

The magnitude of the relationship between the mathematical logical intelligence variable on student learning outcomes in linear algebra courses is 0.726 which is shown in the R=0.726. The magnitude of the influence of the mathematical logical intelligence variable on student learning outcomes is 0.527 or 53% which is shown in the R^2=0.527, while for 47% of the student learning outcomes variable is influenced by other variables outside the study. Thus, this shows that there is an influence of the mathematical logical intelligence variable on student learning outcomes in linear algebra courses.

From the results that have been stated, it can be interpreted that mathematical logical intelligence has an influence of 53% on student learning outcomes in linear algebra courses. This is in line with research conducted by Dafid et al. (2022) which gives the result that mathematical logical intelligence contributes to the improvement of student learning outcomes in Linear Algebra lectures. Milsan and Wewe (2018) obtained the result that there is a significant relationship between mathematical logical intelligence and student learning outcomes. Mujiani (2016) obtained the result that there is an influence between mathematical logical intelligence on student learning outcomes. With this research, it is hoped that educators will be able to apply learning models that can be adapted to mathematical logical intelligence in students, especially in linear algebra courses so that student learning outcomes are more optimal.

CONCLUSION

Based on the research results that have been described, it can be concluded that there is an influence between mathematical logical intelligence on student learning outcomes in Linear Algebra courses of 0.527 or 53%. It is hoped that further research can use this research as a reference, especially research related to mathematical logical intelligence on learning outcomes by adding several relevant variables.

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The Effect of Student Attitudes on Mathematical Understanding Ability in Mathematical Logic in Hybrid Learning

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Abstract

Students' attitudes in mathematical logic courses can have an influence on their learning outcomes. The type of research used is quantitative research with quasi-experimental methods. The sample in this study were third semester students of the informatics engineering study program who took mathematical logic courses totaling 127 students. The results of the research obtained are that there is an effect of attitude on the learning outcomes of student mathematical logic. The results of the regression model obtained with the value of F=7.055 (p-value=0.009<0.05=α), which indicates that there is a significant effect of attitude on the learning outcomes in mathematical logic. The result of the correlation value R is 0.231 and the coefficient of determination (R^2) is 0.053 which shows that the effect of the independent variable (attitude) on the dependent variable (learning outcomes in mathematical logic) is 5.3%. This shows that there is a positive influence of student attitudes on student learning outcomes in mathematical logic.

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INTRODUCTION

After a long pandemic, many things have changed with activities and interactions in many fields. One of them is changes in the field of education. Education that is carried out after passing the pandemic is the implementation of hybrid learning. Learning that is carried out with face-to-face and online meetings is carried out alternately. One of the private campuses in Jakarta is still implementing hybrid learning. Students are asked to be able to attend lectures consistently both offline and online meetings.

Hybrid learning provides a new learning experience for students. This is in line with the opinion by Wegasari, Utomo, and Surachmi (2021) which states that "online learning has a positive impact on students, namely it can provide new experiences. Students are asked to be ready to carry out learning by attending class and carrying out online learning where students are required to be adaptive to technology such as the use of zoom, google meet, and other platforms that support online learning (Sur et al., 2020). Some students have certainly shown positive and negative attitudes during the implementation of hybrid learning. Students' attitudes towards hybrid learning cannot be separated from how students respond to the learning experience they follow.

Attitude is one of the things that is part of the assessment in the learning process in the classroom. Through attitude, lecturers can see the effectiveness of learning in the classroom. Attitude is a reflection of students' interest or disinterest in the course they are taking. Therefore, attitude is an important part of achieving learning objectives in the classroom. The importance of student attitudes in this lecture process is to show the pleasure and seriousness possessed by students in taking mathematical logic courses in hybrid learning.

Basically, attitudes are more consistent than feelings and emotions, but attitudes can affect student activeness in lectures (Idris et al., 2021), but as a predisposition, attitudes cannot be seen or measured directly. Instead, attitudes are inferred from the way we respond to certain stimuli (Sarmah & Puri, 2014). The influence of attitude on students' activities in lectures may be positive or negative depending on the individual students (Mazana, Montero, & Casmir, 2018). Students' attitudes towards learning mathematics can be shown in the form of liking or disliking the course; the tendency of students to participate or not participate in mathematics learning activities; and beliefs about the usefulness or uselessness of mathematics (Kibrislioglu, 2015). Attitude is more about the way a person responds positively or negatively to an object, situation, concept or person. Attitude is a person's belief that reflects opinions and feelings and can be manifested in behavior (Joseph, 2013).

According to Karnia and Nurhasan (2023), attitude is a positive or negative reaction to an object. Furthermore Notoatmodjo (2010) stated that attitude is a collection of symptoms in responding to a stimulus so that it involves thoughts, feelings, attention and other psychological symptoms. The same thing is also stated by Arrosih, Marianti, and Rasidi (2022) that attitude is an internal symptom with an effective dimension in the form of a tendency to respond in a relatively fixed way to objects both positively and negatively. Then Rati, Martias, and Darman (2015) explained that attitude is a readiness to act in a certain way towards an object. Therefore, attitude is one of the factors that influence learning outcomes. According to Syah (2013) stated that attitude is one of the factors that influence the success of learning outcomes (Rati et al., 2015; Djaali, 2012). A positive attitude towards learning is a form of lecturer success in implementing learning in the classroom. This success is also expected to have a positive impact on student learning outcomes.

According to Rati et al. (2015), learning outcomes are cumulative of several aspects that have basically been done by students, both at school and outside school. Furthermore, according to Arrosih et al. (2022), learning outcomes are a process for determining the value of student learning through measuring learning outcomes. According to Dimyati and Mudjiono (2011), learning outcomes are a better level of mental development when compared to before learning. Learning outcomes are certain competencies both cognitive, affective and psychomotor that students achieve after participating in learning (Kunandar, 2014; Rusman, 2016; Sani, 2019; Susanto, 2013). Good learning outcomes are obtained from a series of learning process activities that students follow with a positive attitude.

Some studies that show that a positive attitude will result in good learning outcomes include Trisnawati, Sastrawan, and Buwono (2016) showing that there is an effect of learning attitudes on student learning outcomes by 5.9%. Then Arrosih et al. (2022) stated that there was a positive and significant influence between learning attitudes on student math learning outcomes. One of the positive attitudes seen when students are happy to take part in learning mathematical logic courses.

Mathematical logic is one of the courses that must be followed by third semester students of the informatics engineering study program. Mathematical logic is one of the courses whose material is a lot of symbols, notations and proofs (premises). Therefore, students are asked to be able to prove premises with truth tables by using existing symbols and notations correctly. Therefore, it is necessary for students to have a positive attitude in participating in the course. These positive attitudes include enjoying participating in class learning, doing exercises and assignments, actively participating in class learning, providing ideas, ideas, opinions and even questions during the learning process in class.

Based on previous research conducted by Hartati (2015), it shows that the mathematics learning outcomes of students who have a positive attitude towards mathematics are higher than students who have a negative attitude towards mathematics. The difference in research conducted by researchers is in the research subject. The Hartati's research used a sample of vocational students while the sample in this study was second semester students. In addition, the purpose of the Hartati's research was to see the effect of learning styles and student attitudes on mathematics subjects while the purpose of this study was to see the effect of student attitudes on mathematical logic. Furthermore, the results of research conducted by Purnomo (2017) show that there is a significant effect of attitude on student math learning outcomes. The research subjects conducted by Purnomo were junior high school students while the sample in this study were second semester students. The purpose of Purnomo's research was to see the effect of students' attitudes and learning independence on math learning achievement while this study aims to see the effect of student attitudes on mathematical logic. Then the research conducted by Kibrislioglu (2015) shows that students' positive attitudes arise because of students' love for their teachers, not because of the nature of mathematics. The difference between Kibrislioglu's research and this study is in the research subject. The subject of Kibrislioglu's research was 6th grade elementary school students while the sample of this study was second semester students.

The importance of a positive attitude towards mathematical logic can impact students to learn better. Students who have a positive attitude towards mathematical logic tend to be more motivated to learn. Students have a greater interest in the material and are able to see its importance in everyday life. Then the positive attitude of students can help in overcoming obstacles and difficulties that arise during learning. Students do not give up easily when facing difficult tasks and try their best to achieve a good understanding. Students' positive attitude towards mathematical logic can make students participate more actively such as asking questions, answering questions, providing ideas or responses and discussing with friends in understanding material concepts better. Thus, it is necessary to see whether there is an influence of student attitudes on the learning outcomes in mathematical logic and how much influence student attitudes have on the learning outcomes in mathematical logic. Based on the description above, the researcher is interested in seeing the effect of student attitudes on the learning outcomes in mathematical logic in hybrid learning.

RESEARCH METHODS

This research was conducted at one of the private universities in Jakarta. The research sample is Informatics Engineering students in even semester 2022-2023. Sampling in this study was carried out using Purposive sampling technique, which is a sampling technique based on certain considerations. The research was conducted for one semester from March 2023 to July 2023. This study uses a quantitative approach with the type of cause and effect research that aims to examine the effect of student attitudes (X) on mathematical logic learning outcomes (Y). The research instrument used a questionnaire containing 16 statement items about attitudes distributed to 127 students from four regular morning classes. This questionnaire was developed by Rezeki and Mutia (2020) with indicators (1) showing students' enjoyment in mathematical logic courses; (2) showing students' seriousness in following the mathematical logic course lecture process; (3) showing students' preference for lectures with cooperative learning models; (4) the role of lecturers in lectures. Data were obtained from the results of a questionnaire on the attitude of students learning mathematical logic and 5 items of mathematical logic questions in essay form. The validation of the attitude questionnaire used a Likert scale. The attitude questionnaire was distributed through the class wag in the form of a Google form link.

The analysis used is a simple linear regression analysis technique that aims to determine the effect of student attitudes on mathematical logic learning outcomes and to determine how much influence student attitudes have on mathematical logic learning outcomes.

RESULTS AND DISCUSSION

The descriptive statistical results of students' mathematical logic learning outcomes on student attitudes presented in Table 1.

Table 1. Descriptive statistics results

	N	Min	Max	Mean	Std. Deviation	Variance
Attitude	127	35.00	63.00	50.1575	6.21369	38.610
Learning Outcomes	127	0.00	100	90.2520	11.78105	138.793

In Table 1, it is shown that the minimum value difference between attitude and math logic learning outcomes is 35 while the maximum value difference is 37. Then the maximum and minimum value difference of attitude is 28 while the maximum and minimum value difference of math logic learning outcomes is 100. The difference in value between attitude and learning outcomes is quite large, namely 72. Furthermore, the difference in average value between attitude and math logic learning outcomes is 40.09. This shows that the average value of math logic learning outcomes is much higher than the average value of attitude. The variance results show that the distribution of attitude data is smaller than the distribution of data on math logic learning outcomes. The same thing is also for the comparison between the average value and the standard deviation value, the value of learning mathematical logic is better because it has a smaller standard deviation value than the average value.

Table 2. Anova test results

	Sum of Squares	*df*	Mean Square	*F*	*p*-value
Regression	259.908	1	259.908	7.055	0.009
Residual	4604.942	125	36.840		
Total	4864.850	126			

Table 2 shows that the value of F=7.055 (p-value=0.009<0.05=α), so that the regression model can be used to predict the attitude variable or in other words there is an influence of the attitude variable on the variable learning outcomes in mathematical logic.

Table 3. Regression coefficient

	Unstandardized Coefficients		Standardized Coefficients	*t*	*p*-value
	B	Std. Error	Beta		
(Constant)	68.271	8.338		8.188	0.000
Attitude	0.438	0.165	0.231	2.656	0.009
R	0.231				
R^2	0.053				
Adjusted R^2	0.046				

Table 3 shows that the simple regression coefficient value of math logic learning outcomes is 0.438 while the constant value is 68.271. This can be formulated in the following equation, $\hat{Y}$=68.271+0,438X. This equation can be interpreted that every one unit increase in the attitude variable will also be followed by an increase in the student math logic learning outcomes variable by 0.438. The higher the student's attitude, the higher the student's mathematical logic learning outcomes.

Furthermore, the correlation value R is 0.231 and the coefficient of determination (R^2) of 0.053, which indicates that the effect of the independent variable (attitude) on the dependent variable (mathematical logic) is 5.3%. The results of the above calculations show that there is a positive influence of student attitudes on student mathematical logic learning outcomes. Students who have a high attitude tend to show interest and effort to participate in the learning process of mathematical logic. Conversely, students who have a low attitude will tend to show less interest and less participation in the learning process of mathematical logic. This is in line with the opinion Hartati (2015) which states that if students have a positive attitude towards the course, these students will categorize the course as an interesting and useful course to study, such as enjoying doing assignments, responding well to problems and challenges given by lecturers and so on and vice versa.

Based on the research results, the coefficient of determination is 5.3%. This shows that the learning outcomes of student mathematical logic are influenced by student attitudes. Students' environmental factors are one of the reasons for this. When given a task, each group tries to complete the task in a timely manner. However, the factor of homogeneous group members is one of the things that influences the process of group discussion activities. Groups consisting of students with moderate abilities will show a positive attitude in completing the task. Each student helps each other to complete the task. Conversely, groups consisting of low-ability students will show negative attitudes such as laziness to try to solve problems, pessimistic about being able to complete the task and hope that friends can complete the task.

Student Attitudes Positively Impact Math Understanding

Based on the results of the analysis test, it can be shown that there is a positive effect of student attitudes on student learning outcomes in mathematical logic courses. Students who have a positive attitude tend to have good learning outcomes in mathematical logic courses. This can be seen from the results of the coefficient of determination (R^2) which shows that the magnitude of the influence of student attitudes on learning outcomes in mathematical logic courses is 5.3%. In other words, 5.3% of student learning outcomes are influenced by the attitude of the students themselves and the rest is influenced by other factors. Among other factors that affect student learning outcomes in mathematical logic courses is the division of groups homogeneously. Homogeneous group division based on the value of the prerequisite courses in the previous semester. Groups consisting of students whose prerequisite scores are low will show negative attitudes such as being less active in lectures, less motivated in lectures, late in submitting assignments and less serious in attending lectures. Meanwhile, groups consisting of students whose prerequisite scores are high will show positive attitudes such as being happy to attend lectures, active in group discussions, on time in submitting assignments and having high curiosity about mathematical logic courses. It is also explained by Hartati (2015) that if students have a positive attitude towards the course, these students will categorize the course as an interesting and useful course to learn, such as happy to do assignments, respond well to problems and challenges given by lecturers and so on. Conversely, students who have a negative attitude towards the course will categorize the course as an uninteresting and less useful course to learn, such as being forced to do assignments, lazy to do problems and so on.

Although the percentage is small, it still has a positive influence on student learning outcomes. The same thing was also obtained from the results of research conducted by Trisnawati et al. (2016) which showed that there was an effect of learning attitudes on student learning outcomes of 5.9%. According to Purnomo (2017) states that students who have a positive attitude will show a sense of pleasure and a greater desire to get to know and know more about mathematical concepts. In line with the opinion by Hartati (2015) which also states that students will take sustainable actions if they feel happy doing something, such as studying, attending lectures and so on. Based on this opinion, it can be concluded that a positive attitude is very important for every student in attending lectures in order to obtain good learning outcomes.

Based on the results of the anova test, the value of *F*=7.055 (*p*-value=0.009 <0.05=α), this shows that student attitudes have a significant influence on learning outcomes in mathematical logic courses. This is shown during class lectures that some students are active in answering questions and questions given by the lecturer. During group discussions, it was seen that some students participated in providing ideas and ideas in solving exercise problems. In addition, some students also have the initiative to present the results of their group discussions in front of the class. This is in line with the opinion by Purnomo (2017) which states that the positive attitude of students in the course can have a good influence on learning outcomes. Furthermore, according to Aisah (2023) states that the attitude of students who play an active role during lectures can provide good learning outcomes. Conversely, the attitude of students who do not play an active role during lectures can provide poor learning outcomes. The opinions of the two experts are corroborated by the results of research conducted by Arrosih et al. (2022) showing that there is a positive and significant influence between learning attitudes on student math learning outcomes.

CONCLUSION

The results showed that there was an effect of attitude on the learning outcomes of student mathematical logic. It is interpreted that the higher the student's attitude, the higher the student's mathematical logic learning outcomes. The effect of attitude on students' mathematical logic learning outcomes is 5.3% and the rest is influenced by other factors.

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Improving Students' Mathematical Problem Solving Ability in Trigonometry Material Using the Flipped Classroom Model

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Abstract

This research was motivated by the relatively low learning outcomes and problem solving abilities of class IX students. So this research aims to improve learning outcomes and improve students' mathematical problem solving abilities. The objectives were achieved through the classroom action research method by applying the flipped classroom model for 3 cycles in trigonometry learning. This research involved 32 students from SMA N 1 Pedes. Data was collected using test instruments, observation sheets, and interviews. Data were analyzed using the Miles & Hubberman method. The results of the analysis show an increase in learning outcomes and an increase in mathematical problem solving abilities. The increase in learning outcomes is shown by the average value in cycle I of 67.28, cycle II of 72.97, and cycle III of 75.41. The percentage of mathematical problem solving ability from cycle I was 47%, increasing in cycle II to 71.87%; and in cycle III it was 84.37%. Thus, student learning outcomes have reached the specified criteria of 74, and meet the criteria for the percentage of problem solving ability set at 80%. So the application of the flipped classroom model can be recommended for further use in mathematics learning, especially trigonometry.

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INTRODUCTION

Mathematics is taught to students in Indonesia from elementary to secondary level, one of the goals is to equip students with problem solving skills (Kemendiknas, 2010). Through the learning process students will practice solving mathematical problems which will be useful when they are carrying out the process of integrating mathematics into other fields or when implementing it into everyday life problems (English & Sriraman, 2010). This goal is also in line with the mathematics learning outcomes standard from the National Council of Teachers of Mathematics (NCTM) which states that students are able to solve everyday problems with the help of the mathematical concepts and procedures they have learned (Bossé et al., 2010). It was also stated that strengthening the attitude of perseverance and never giving up for students during the process of solving mathematical problems is part of the cultivation of positive national character that needs to be continuously developed (Almerico, 2014; Freeman-Green et al., 2015; Rokhman et al., 2014). So strengthening problem-solving abilities in students through learning mathematics is very important. Previous experts such as Dewey (1916) and Bruner (1961) have also argued about the importance of an educational process that is oriented towards the formation of problem-solving competencies.

Unfortunately, the ability to solve mathematical problems which is the target of mathematics learning in schools in Indonesia is not yet fully as expected. The results of the analysis of problem solving abilities carried out by previous researchers show that students' mathematical problem solving abilities at several levels of Senior High School are still in the low category (Akbar et al., 2018; Nugraha & Zanthy, 2019; Rachmawati & Adirakasiwi, 2021). Therefore, researchers are challenged to participate in efforts to improve the problem-solving abilities of students in Indonesia, the aim of which is none other than to ensure that the quality of graduates is increasingly high quality. This is one of the reasons why this research needed to be carried out. To find out students' mathematical problem solving abilities in more detail, the researchers made observations at one of the high schools in Karawang, Indonesia. Based on the results of observations at SMA N 1 Pedes, the researchers obtained data on mathematical problem solving abilities that were still relatively low in class IX. This is shown by the results of the Final School Examination where only 20% of students succeeded in achieving the Minimum Completeness Criteria. To confirm the students' mathematical problem solving abilities in this class, the researchers gave pretest questions. The pretest results are presented in Figure 1.

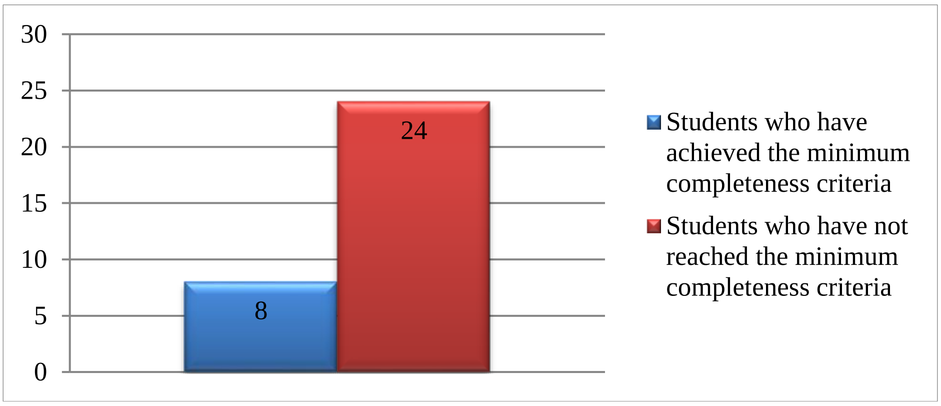

Figure 1. Pretest results of mathematical problem solving ability

Through the pretest results shown in Figure 1, it can be seen that only 8 students or around 25% of the 32 students succeeded in reaching the Minimum Completeness Criteria. So, through the results of this pre-test, it can be said that the students' mathematical problem solving abilities in this class are really low.

The researcher then carried out further observations to find out the factors causing the low mathematical problem solving abilities in this class. Based on observations, it is known that the learning usually carried out by teachers has not integrated certain learning models with an orientation towards increasing mathematical problem solving abilities. Another finding is that students are not yet motivated to be actively involved in problem solving from the contextual questions presented. Based on the mathematics learning problems found, researchers were encouraged to get involved in looking for alternative solutions. If it is based on the causal factors found, a learning process is needed that can encourage students to be actively involved in the process of solving mathematical problems. Nurafifah, Nurlaelah, and Usdiyana (2016), argue that if teachers are willing to utilize certain appropriate learning models it will have a positive influence on students' mastery of mathematical problem solving. A similar opinion also states that the use of models that are appropriate to the learning objectives to be achieved will have a positive impact on student learning outcomes (Alifah, 2019; Fahrurrozi, Mohzana, & Murcahyanto, 2021; Khoerunnisa & Aqwal, 2020; Warsita, 2018).

As an alternative solution, it is necessary to improve learning methods from the methods previously used by teachers. Before determining the learning method to be used, it is necessary to remember that the current implementation of the Independent Curriculum requires the integration of technology in the learning process (Cholilah et al., 2023; Nuridayanti et al., 2023; Widiyono & Millati, 2021). Thus, the learning method that is intended to be used as an alternative solution needs to consider the objective aspect, namely increasing student activity in solving mathematical problems, and the aspect of using technology to accommodate the demands of implementing The Independent Curriculum.

Based on the literature, one learning model that can increase student activity and at the same time involve the use of technology is the flipped classroom model. The flipped classroom is a learning model that uses multimedia devices and technology to help exchange time for delivering learning material (Hwang & Lai, 2017), so that students receive the most support when they are working on assignments that require a lot of additional theory while in class (Lai & Hwang, 2016). Apart from that, the flipped classroom model is the right strategy and is in accordance with the development of 21st century learning technology (Widyasari, Masykur, & Sugiharta, 2021). The flipped classroom model is recommended for use in this research based on several opinions which state that there are benefits in line with the objectives of this research. This benefit is that it can maximize student activity through the process of interaction between students and teachers, and between students and each other, which can encourage an increase in students' mathematical problem solving abilities (Rohmatulloh & Nindiasari, 2021; Threlkeld, 2017; Widyasari et al., 2021; Zainuddin & Attaran, 2016). The steps for the flipped classroom learning model that will be used are adapted from Pardimin et al. (2020) which consists of learning outside the classroom and inside the classroom. Sequence of learning steps outside the classroom consists of: (1) the teacher shares learning videos via WhatsApp group; (2) students study the material by listening to the video; (3) students record the problems they find. As for the class, it consists of: (1) the teacher asks about the material that has been studied through video; (2) the teacher asks if there is any material that the students have not understood; (3) the teacher explains the material that the students have not understood; (4) the teacher gives practice questions to the students; (5) the teacher gives instructions for students to solve the questions, and; (6) the teacher monitors the student discussion process. Next, the learning steps will be applied to learning trigonometry material. Trigonometry material was chosen because it corresponds to the target material that must be taught in the curriculum, and is based on analysis results which show that high school students still experience errors when solving trigonometry problems (Setiana, Fitriani, & Amelia, 2021; Tunnajach & Gunawan, 2021).

In accordance with the background of the problem that has been explained, after being given treatment in learning trigonometry using the steps of the flipped classroom learning model, it is hoped that students' learning outcomes and mathematical problem solving abilities will increase. Learning outcomes are said to improve if there is an improvement in the average value obtained after being given treatment. Students' mathematical problem solving abilities are said to increase if they fulfill the indicators from Polya (1985): (1) understanding the problem; (2) planning the solution; (3) resolve problems according to plan, and; (4) recheck the results obtained.

RESEARCH METHODS

This research is in the form of Classroom Action Research, namely research conducted on learning activities which are deliberately raised and actions carried out together in a class (Arikunto, 2021). The purpose of doing class action in this study was to improve the learning process with the ultimate goal of improving the mathematical problem solving abilities of class XI students. The class action carried out was in the form of applying the flipped classroom learning model to trigonometry material which consisted of three cycles. Each class action cycle is carried out following the procedure developed by (Arikunto, 2021) as shown in Figure 2.

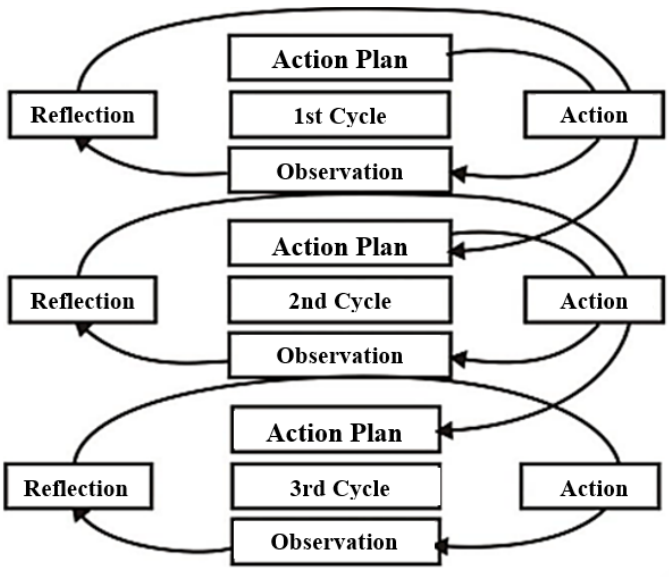

Figure 2. Classroom research model by Arikunto (2021)

It can be seen from Figure 2 that the classroom action research procedure carried out consists of the following stages: (1) planning; (2) action; (3) observation, and; (4) reflection. Before the class action was carried out, the researcher first carried out the pre-action stage which aimed to divide the learning material into the three planned cycles, as well as establish the initial conditions as the basis for measuring an increase in mathematical problem-solving abilities.

This classroom action research involved 32 students who were in one class and the researcher himself acted as a teacher. The other party involved is the senior mathematics teacher at the school who acts as an observer. Research data were collected using written test instruments, observation, interviews, and documentation. The developed written test instrument contains indicators of problem-solving abilities from Polya (1985), which consist of: (1) understanding the problem; (2) determine a strategic plan for problem solving; (3) complete the strategic plan, and; (4) recheck answers. Observations were made using an observation sheet which contained indicators: (1) students focused on problems; (2) students are able to describe the sketch of the problem; (3) students carry out planning for completion; (4) students carry out plans, and; (5) evaluation of results. The interviews were conducted using the indicators contained in the interview guidelines. The data obtained was then processed using the method by Miles and Hubberman (1984) until a conclusion was drawn. The criteria for the success of the learning process determined in this research are if 80% of students have achieved the Minimum Completeness Criteria and the percentage of students' mathematical problem solving ability achievement is 75%.

RESULTS AND DISCUSSION

Before implementing the class action stage, a pre-action stage is first carried out which consists of determining the initial conditions and distributing the material to be taught in each cycle. Initial conditions were determined by providing pretest questions, the results of which are presented in Figure 1. Thus, the initial conditions before action was taken showed that only 8 students (25%) passed the Minimum Completeness Criteria. After going through 3 cycles of classroom action, it is assumed that students who successfully achieve the KKM will improve, followed by an increase in students' mathematical problem solving abilities, especially in trigonometry material. The trigonometry material is divided into 3 cycles, namely cycle 1 and cycle 2 with the subject of trigonometric equations, and cycle 3 with the subject of trigonometric equations involving the sum and difference of angles. Next, class action was carried out, the results of which were as explained below.

Class Action Results in Cycle 1

Cycle 1 was held in 3 meetings, each meeting was held in 2 class hours. The first meeting on May 4 2023 held a discussion regarding the learning process plan, and the material on trigonometric equations was explained. The second meeting was held on May 5 2023 and carried out further discussions regarding trigonometric equations. The 3rd meeting on May 11 2023 carried out a posttest. The results of class actions from each stage in cycle 1 are explained as follows.

Results of the Planning Stage

The results of the planning stages are: (1) lesson plan with limited materials on trigonometric equations; (2) develop a cycle 1 posttest instrument consisting of 5 questions; (3) compiling observation sheets; (4) compiling an interview sheet; (5) dividing student groups according to the needs of implementing the flipped classroom model, and; (6) prepare a learning video on the subject of basic trigonometry equations in the video link: https://www.youtube.com/watch?v=l5ec3hUNWSQ.

Results of Implementation Phase

At the first meeting the researcher started the learning activity by informing the results of the division of the study group which was divided into 6 groups consisting of 5 to 6 students. Next, the researcher provided a link to the learning video to be studied at home and used as material for group discussion at the next meeting. At the second meeting, a group discussion was held regarding the concepts of sine, cosine and tan equations, as contained in the video. In general, the learning process at the first and second meetings ran smoothly, and was attended by observers who were tasked with observing teacher and student activities during the lesson. At the third meeting, a posttest was carried out which was attended by 32 people, the results of which can be seen in Table 1.

Table 1. Cycle 1 posttest results

Achievement	The number of students	%	Average
Pass Criteria	15	47%	67,25
Did’n Pass Criteria	17	53%	

Table 1 shows that 47% of students achieved the Minimum Completeness Criteria score and 53% of students did not complete it. Because the success criteria set is that 80% of students must complete the Minimum Completeness Criteria, it can be said that students’ learning outcomes in cycle 1 are still low and improvements need to be made in the next action process.

Observation Phase Results

Based on the observation sheet, it can be shown that the percentage of students' mathematical problem solving abilities in cycle 1 was 61.17%. Because the success criteria are set at 75%, it can be said that the achievement of students' mathematical solving abilities at this stage is still low. Based on the results of the observations, it was found that students lacked focus, which was indicated by behavior that disturbed other students, and it was also found that students felt anxious when they did not understand the problem-solving steps that should be taken. Based on the results of the observer's observations, it was found that teacher activities were not optimal, so it was recommended that their activities be increased.

Results of the Reflection Stage

Based on the observation results, it was found how many things needed to be improved in cycle 2, including teachers needing to increase activities aimed at making students focus more on the learning process. Apart from that, teachers need to take a personal approach to students who are seen as experiencing mathematical anxiety.

Class Action Results in Cycle 2

Cycle 2 was held in 3 meetings, where each meeting was held in 2 class hours. The first meeting on May 12 2023 was held to discuss the learning process that would be carried out and discussed the material on trigonometric equations. The second meeting on May 17 2023 held a further discussion regarding trigonometric equations. The third meeting will be held on May 19 2023, where the posttest will be held. The results of each stage of class action carried out in cycle 2 are explained as follows.

Results of the Planning Stage

The results of the planning stages are: (1) lesson plan with limited material on trigonometric equations; (2) develop a cycle 1 posttest instrument consisting of 5 questions; (3) compiling observation sheets; (4) compiling an interview sheet; (5) dividing student groups according to the needs of implementing the flipped classroom model, and; (6) prepare a learning video on the topic of trigonometric equations involving trigonometric comparisons with the video link: https://www.youtube.com/watch?v=Yk4SmyMqobE.

The teacher's improvement plan to direct students to be more focused is to prepare several rhymes related to trigonometry material. One of the verses of the prepared rhyme reads: *“Pergi ke pasar untuk beli ikan teri, pulangnya ketemu bekantan. Ayo belajar trigonometri, karna sudah ditunggu sin, cos, dan tan.”* Next, the teacher plans to take a personal approach to students who experience mathematics anxiety, namely by providing motivation and suggesting more discussions with their colleagues.

Results of Implementation Phase

At the first meeting, the researcher started the learning activities by informing the results of the posttest from cycle 1. Next, the teacher provided a link to the learning video to be studied at home and used as material for group discussion at the next meeting. At the second meeting, the second meeting carried out learning activities using group discussions. At the second meeting A group discussion was held regarding trigonometric comparisons. The learning process generally went smoothly and was attended again by observers. At the third meeting, a posttest was carried out which was attended by 32 students, with the results which can be seen in Table 2.

Table 2. Cycle 2 posttest results

Achievement	The number of students	%	Average
Pass Criteria	23	71,87%	72,97
Did’n Pass Criteria	9	28,13%	

Table 2 shows that 71.87% of students achieved the Minimum Completeness Criteria score, while 28.13% of students did not complete it. Thus, student learning outcomes in cycle 2 cannot be said to be successful because they have not been able to reach the success criteria, namely 80% of students must complete the Minimum Completeness Criteria.

Results of the Observation Phase

Based on the observation sheet, it can be shown that the percentage of students' mathematical problem solving ability increased to 74.17%, thus the action process in cycle 2 cannot be said to have succeeded in reaching the set criteria for the percentage of problem solving ability, namely 75%. Based on the results of observations, it is known that some students' weaknesses in the process of solving mathematical problems are that they still experience difficulties in physically describing problems and planning solutions. Based on the observer's observations, it can be shown that teacher activities have been optimally improved, but teachers still need to provide intensive assistance to several students who experience difficulties when writing forms of mathematical problem solving plans.

Results of the Reflection Stage

Based on the observation results, several things were found that needed to be improved when implementing actions in cycle 3, including researchers needing to provide intensive assistance to several students who were still experiencing difficulties in formulating problem-solving steps.

Class Action Results in Cycle 3

Cycle 3 was held in 3 meetings, each meeting being held in 2 class hours. The first meeting on May 25 2023 held a discussion on trigonometric sum and difference equations. The second meeting on May 26 2023 held a further discussion regarding trigonometric sum and difference equations. The third meeting on May 31 2023 held a posttest. The results of the actions from each stage in cycle 3 are explained as follows.

Results of the Planning Stage

The results of the planning stages are: (1) lesson plan with limited material on trigonometric sum and difference equations; (2) develop a cycle 1 posttest instrument consisting of 5 questions; (3) compiling observation sheets; (4) compiling interview sheets, (5) dividing student groups according to the needs of implementing the flipped classroom model, and; (6) prepare a learning video on the subject of trigonometric equations of quantities and angle differences with the video link: https://www.youtube.com/watch?v=AEnPOa4hubw.

The teacher's plan for students who still have difficulty making problem solving plans is to remind them of the problem solving stages from Polya (1985). Next, the teacher will provide special assistance to students who are still experiencing the same difficulties.

Results of Implementation Phase

At the first meeting, the researcher started the learning activity by informing the results of the posttest from stage 2. Next, they provided a learning video link with material on trigonometric sum and difference equations to be studied at home and used as material for group discussion. At the second meeting, group discussion activities were carried out discussing trigonometric sum and difference equations. Next, at the third meeting, a posttest was carried out followed by 32 students, the results of which are presented in Table 3.

Table 3. Cycle 3 posttest results

Achievement	The number of students	%	Average
Pass Criteria	27	84,37%	75,41
Did'n Pass Criteria	5	15,63%	

Table 3 shows that 84.37% of students have reached the Minimum Completeness Criteria, while 15.63% of other students have not yet completed it. If it refers to the set criteria, the learning process in cycle 3 can be said to have succeeded in achieving the set criteria.

Observation Phase Results

Based on the observation sheet, it can be shown that the percentage of students' problem solving abilities has increased to 81.83%. Thus, it can be said that the increase in students' mathematical problem solving abilities has reached the expected criteria.

Results of the Reflection Stage

Based on the achievement of learning outcomes and the increase in the percentage of students' mathematical problem solving abilities in cycle 3, it was decided that this classroom action research did not need to be continued in the cycle, and this research was considered successful.

Classroom action research which was planned to be carried out in 3 cycles has been completed according to established procedures. In general, the class action process in the form of implementing the flipped classroom model in class XI in learning trigonometry has been carried out without problems. Based on the results of the posttest in cycle 1, it is known that only 47% of students whose learning outcomes reached the Minimum Completeness Criteria, and the percentage of problem solving ability achieved was 61.17%. Next, the action process in cycle 2 is carried out based on suggestions for improvement provided by the observer. Based on the results of the posttest in cycle 2, it can be shown that 71.87% of students achieved the Minimum Completeness Criteria with a percentage of problem solving ability achievement of 74.17%. Learning in cycle 3 was carried out again by following suggestions from the observer. Based on the results of the posttest in cycle 3, it can be shown that student learning results have achieved the Minimum Completeness Criteria of 84.37% with a problem solving ability percentage of 81.83%. In general, the increase in student learning outcomes is presented in Figure 3.

Based on Figure 3, researchers have been able to show that there is an increase in learning outcomes in each cycle, namely from the pre-cycle stage to cycle 1 it increased by 22%, then from cycle 1 to cycle 2 it increased by 24.80%, and from cycle 2 to cycle 3 increased again by 12.57%. Based on Figure 3, it can also be shown that the student learning outcomes in cycle 3 have reached the specified criteria, namely that there are 27 students or around 84.37% who have reached the Minimum Completeness Criteria. This concludes that the action process taken has had a positive impact on efforts to improve learning which is the aim of this research.

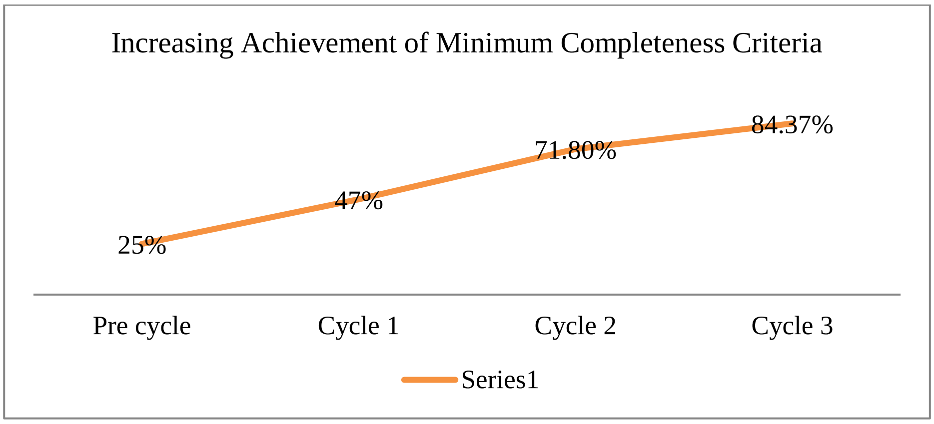

Figure 3. Improved learning outcomes in each cycle

Furthermore, based on the results of the analysis of the observation sheet, it can be shown that there is an increase in the percentage of students' achievement of mathematical problem solving abilities. The increase in the percentage achievement of students' mathematical problem solving abilities from each cycle is presented in Figure 4.

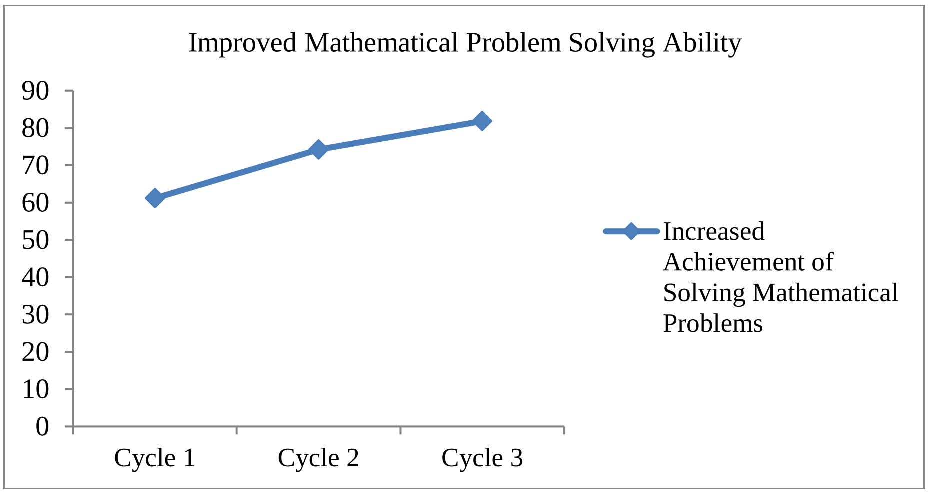

Figure 4. Percentage increase in students' mathematical problem solving ability

Based on Figure 4, it can be shown that there is an increase in the percentage of mathematical problem solving abilities from cycles 1 to 3. Based on Figure 4, it is known that the percentage increase in students' mathematical problem solving abilities has reached 81.83% in cycle 3, so that it has met the success criteria to be achieved in this research. namely 75%. If detailed further, the percentage increase from cycle 1 to cycle 2 increased by 13%, and from cycle 2 to cycle 3 it increased again by 7.66%.

Based on the description above, researchers have been able to show that providing classroom actions through the application of the flipped classroom model in trigonometry material has succeeded in improving learning outcomes and also improving students' mathematical problem solving abilities. The results of this research are in line with the results of previous research, including research conducted by Widyasari et al. (2021) which stated that there was an increase in critical mathematical thinking skills and learning motivation among Madrasah Tsanawiyah students in learning using the flipped classroom. Meanwhile, Saputra and Mujib (2018) research stated that students' understanding of mathematical concepts using the flipped classroom learning model was better than their ability to understand concepts using the lecture learning method.

CONCLUSION

Based on the results of data analysis researchers have been able to show an increase in student learning outcomes and an increase in problem solving abilities. The increase in learning outcomes is shown by the results of the posttest, namely: (1) the average value of learning outcomes in cycle I is 67.28; (2) in cycle II it was obtained at 72.97, and; (3) in the third cycle of 75.41. While the increase in problem solving ability is shown through the observation results sheet, namely: (1) problem solving ability in the first cycle of 47%; (2) in cycle II increased by 71.87%, and; (3) in cycle III it increased again by 84.37%. These results indicate that student learning outcomes have met the specified criteria, specifically achieving a score of 74 and meeting the criteria for problem-solving ability at 80%. Therefore, it can be concluded that the classroom action, in the form of applying the flipped classroom model, improves learning outcomes and enhances students' mathematical problem-solving abilities, particularly in trigonometry. The conducted research has successfully achieved its stated objectives, suggesting that the application of the flipped classroom model is recommended for subsequent years in trigonometry education.

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A Note on Leonardo's Combinatorial Approach

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Abstract

The purpose of this research is to carry out a study of Leonardo's combinatorial approach so that it is possible to visualize these numbers through combinatorial interpretation. Thus, research is being developed regarding methods and approaches to linear and recurring sequences, based on the combinatorial study of the Fibonacci sequence. In fact, the Fibonacci sPquence is related to other sequences, one of which is the Leonardo sequence, which has similarities with the Fibonacci numbers according to some researchers in the field. Given this scenario, the present research addresses the combinatorial interpretation of Leonardo's sequence, allowing the definition of Leonardo's combinatorial model, considering the notion of board and bracelets in Lucas' sequence. As research results, the study deals with the integration of sequence content with the area of Combinatorial Analysis, allowing a mathematical advancement of Leonardo's sequence. Furthermore, you can visualize the sequence numbers in front of the tiles. The aspects studied in this research are linked to the teaching of sequences in the History of Mathematics, allowing the teaching of Mathematics.

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INTRODUCTION

The Fibonacci sequence was created by mathematician Leonardo Pisano (1180-1250) known as Fibonacci or "son of Bonaccio." Born in Pisa, Italy, Leonardo acquired mathematical knowledge of the Arab world and in the areas of Algebra and Arithmetic, being remembered because of the problem of reproduction of immortal rabbits, thus generating the Fibonacci sequence (Santos, 2017). Fibonacci wrote 5 works, among which one of the proposed problems stands out, which marks the genesis of the Fibonacci method: "How many pairs of rabbits will be produced in a year, starting with a single pair, if each month each pair generates a new pair that becomes productive from the second month onwards, as long as no rabbit dies during this period?” of the Santos (2017). When answering this question there is a sequence of numbers, a sequence called a sequence of numbers.

Faced with this question, the Fibonacci sequence can be presented, where having the recurrence formula given by $F_n = F_{n-1} + F_{n-2}$, $n \geq 2$ and n belongs to the set $\mathbb{N}$ and $F_0 = F_1 = 1$.

This sequence inspired other sequences, highlighting Leonardo's sequence for this research. Leonardo's sequel, has few works referring to these numbers found in the literature, therefore highlighting the works of Catarino and Borges (2019), Shannon (2019) and Vieira et al. (2019), in which they define these numbers as second order. In these researches, the absence in relation to the historical process of these numbers is notable, reporting only its mathematical evolution. In fact, no research has been found confirming the creator of this sequence, however, it is believed that, as it has the name of Leonardo's sequence and great mathematical similarities, it was created by mathematician Leonardo Pisano (1180-1250), the same creator of the Fibonacci sequence. Comparing it with the sequence of Fibonacci, we can see that the value 1 was added at the end of the recurrence $Le_n = Le_{n-1} + Le_{n-2} + 1$, $n \geq 2$ and n belongs to the set $\mathbb{N}$ and $Le_0 = Le_1 = 1$.

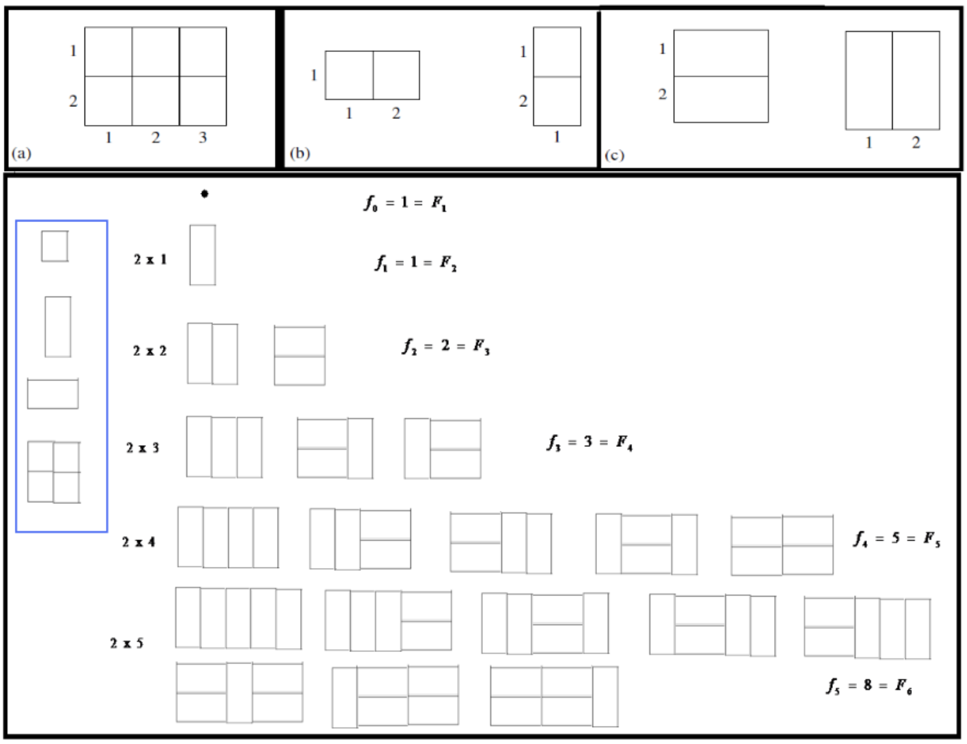
The numerical properties of these numbers are analogous to the Fibonacci properties. Thus, Catarino and Borges (2019) established another recurrence relationship, where from recurrence as being $Le_n = 2\ Le_{n-1} + Le_{n-3}$, $n \geq 3$. The characteristic equation of this sequence can be written based on this recurrence, transforming it into a third-order sequence order. Catarino and Borges (2019) also established a relationship between Leonardo's numbers and Fibonacci, being this $Le_n = 2\ F_n - 1$. So we have $x^3 - 2x + 1 = 0$ presenting three real roots, one being equal to 1 and the other two equal to the roots of the characteristic Fibonacci equation, highlighting the presence of the golden number (value approximately 1.61) as a result of one of the positive roots. Given the brief discussion around the Fibonacci and Leonardo sequences, we have the study of the combinatorial model of Leonardo.

In the context of education, it can be noted that the visualization of the terms in the sequence allows a better understanding of the content, becoming a facilitator of the teaching process. In this way, he was motivated by the study of Leonardo's sequence in light of its combinatorial interpretation.

This passage discusses the historical significance of the Fibonacci Sequence in mathematics and highlights the use of combinatorial interpretations in Elementary Mathematics. It emphasizes the connection between the Fibonacci Sequence and tiling and decomposition of integers. Additionally, it mentions the involvement of prominent researchers in this field (Alves & Sousa, 2023).

Figure 1 deals with a Fibonacci combinatorial approach, using the $2\times n$ board. With this, pieces are available so that you can fill the board and visualize the numbers in the Fibonacci sequence.

Furthermore, the integration between sequences and Combinatorial Analysis can enrich the teaching of other numerical sequences, providing students with a solid foundation to understand and explore different types of sequences and their combinatorial properties. This interdisciplinary approach can be a promising way to promote a more comprehensive and meaningful teaching of numerical sequences, preparing students to face diverse mathematical challenges.

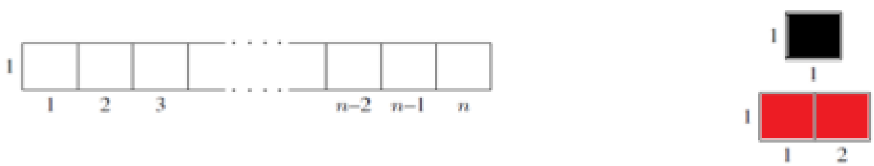


Source: Alves and Sousa (2023)

Figure 1. One interpretation of Fibonacci

FIBONACCI COMBINATORIAL INTERPRETATION

In a generalist way, it is possible to verify contemporary research related to the study of numerical sequences recurring elements and their countless generalizations, which tend to be neglected in books on the History of Mathematics (Grimaldi, 2012; LaGrange, 2013; Stillwell, 2010) An interesting approach is the combinatorial interpretation of the Fibonacci sequence via tiles. So, based on the recurrence of the Fibonacci sequence, given by $F_n = F_{n-1} + F_{n-2}$, $n \geq 2$, whose initial values are given by $F_0 = F_1 = 1$. Thus, we have a $1{\times}n$ type board, with two types of tiles: one $1{\times}1$ black tile and a $1{\times}2$ red tile (dominoes). This one-dimensional n–board is indicated in Figure 2.



Source: Benjamin and Quinn (2003a)

Figure 2. Interpretation of the notion of n–board

Definition 2.1: A board is made up of squares called houses, cells or positions. These positions are enumerated and these enumerations describe the position. A given board will just be called an n–board Spreafico (2014).

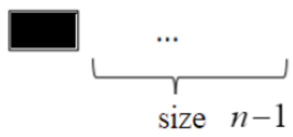
Theorem 2.2: The number of ways to cover an n–board with 1×1 squares and 1×2 dominoes is equal to $f_n = F_{n+1}$ (Spivey, 2019).

Proof. Let S_n be the sum of the tile decks, where $S_0 = 1 = F_1$, $S_1 = 1 = F_2$.

Considering an arbitrary tiling of size n, where $n \geq 2$, we have:

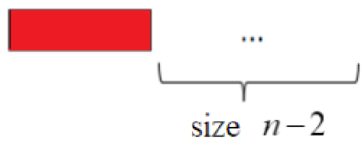
Case 1: Assuming that the tiling starts with the square. These pieces are made up of a square of size 1×1, followed by a screen of size $n-1$ (Figure 3).

The sum is given by S_{n-1}.



Source: Prepared by the authors
Figure 3. Board of size $n-1$

Case 2: Assuming that the tiling starts with a domino of size 1×2. The pieces consist of a board of size $n-2$ and a domino (Figure 4).



Source: Prepared by the authors
Figure 4. Board of size $n-2$

The sum is given by S_{n-2}.

Therefore, according to the principle of addition of the analyzed cases, $S_n = S_{n-1} + S_{n-2}$. Satisfying the Fibonacci recurrence ($F_{n+1} = F_n - F_{n-1}$), with initial conditions $S_0 = F_0$ and $S_1 = F_1$. S_0, $S_n = F_{n+1}$.

In all, the research is based on the works of Benjamin and Quinn (2003b, 2003a), Spreafico (2014) and Koshy (2019), of which describe the notion of an n–board, as well as definitions of terms and expedients to be used in this research, following with the combinatorial properties of the Fibonacci sequence.

Figure 5 shows the cases in the Fibonacci combinatorial model in another format, where 1 represents the square and 2 represents the domino.

Based on the studies carried out on the Fibonacci combinatorial model, we have investigated Leonardo's combinatorics approach. It's worth noting that these connections are often overlooked in specialized History of Mathematics textbooks and are seldom discussed in mathematics teacher training, as mentioned by De-Temple and Webb (2014), Koshy (2019), Spivey (2019) and Vorobiev (2000).

$f_1 = 1$	$f_2 = 2$	$f_3 = 3$	$f_4 = 5$	$f_5 = 8$	$f_6 = 13$		$f_7 = 21$	
1	11	111	1111	11111	111111	1221	1111111	12112
	2	12	112	1112	11112	1122	111112	21112
		21	121	1121	11121	2112	111121	12121
			211	1211	11211	2121	111211	21211
			22	2111	12111	1212	112111	12112
				122	21111	222	121111	12211
				212	2211		211111	11221
				221			11122	21112
							11212	1222
							2221	2122
								2212
$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$		$n = 7$	

Source: Benjamin and Quinn (2003b)

Figure 5. Fibonacci's combinatorial model

A COMBINATORIAL APPROACH TO LEONARDO NUMBERS

In this section, we will follow the discussions highlighted by Koshy (2019) and Benjamin and Quinn (2003b) in which they analyze the combinatorial behavior of the Fibonacci sequence via tiling. Still, we have the relationship established by Catarino and Borges (2019), given by: $Le_n = 2Le_{n-1} - Le_{n-3}$, $n \geq 3$.

With this, we have Leonardo's combinatorial model through tiling and using white squares and dominoes grays and blacks to compose this model. From these available pieces, it is possible to establish a combinatorial interpretation for Leonardo's numbers. The set of objects that are considered is based on tiles of a size range n of squares, that is, a line of n squares.

Let ln be the number of tile shapes on the board of size n and the strip be a board of size n, The following pieces are available: white squares of size 1×1, gray dominoes and black dominoes of size 1×2.

Knowing that gray dominoes and black dominoes cannot occupy the same board, and can only appear alone or with squares.

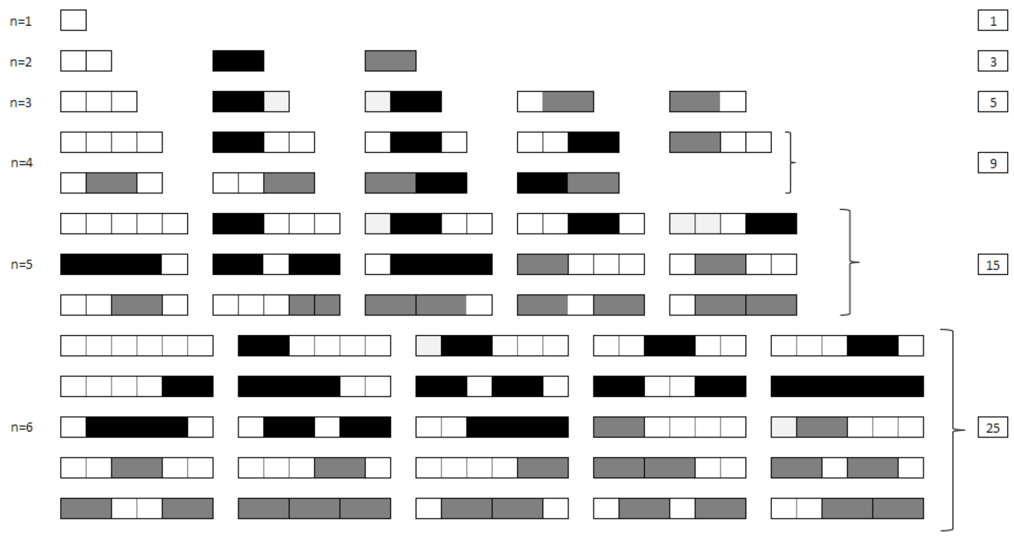
Theorem 3.1: For $n \geq 1$, the possible tiling of a $1 \times n$ board, with white squares and gray dominoes and black dominoes, is given by:

$$le_n = Le_n,$$

where le_n is the number of ways to fill the $1 \times n$ board and Le_n is the nth term of the Leonardo sequence.

Proof. Carrying out the demonstration by dividing the set len into subsets, we have that: Assuming that $n \geq 3$ and divide into two subsets: Q those that start with squares and D those that start with dominoes (red or blue). Like this $le_n = Q \cup D$ and $|\ le_n\ | = |\ Q\ | + |\ D\ |$. Soon $|\ Q\ | = le_{n-1}$ and $|\ D\ | = le_{n-1} - le_{n-3}$, resulting in $le_n = |\ Q\ | + |\ D\ | = le_{n-1} + le_{n-1} - le_{n-3} = 2\ le_{n-1} - le_{n-3}$. Therefore, le_n satisfies the same recurrence formula with initial conditions of Le_n. Therefore, $le_n = Le_n$, for everyone $n \geq 1$.

For better understanding of the reader, Figure 6 shows Leonardo's combinatorial model for the case $n = 1$, with 1 form of tiling. For the case $n = 2$, there are 3 ways of tiling. For n = 3, there are 5 ways of tiling. For n = 4, we have 9 ways of tiling. For n = 5, there are 15 ways to tile. For n = 6, there are 25 ways to tile.



Source: Prepared by the authors

Figure 6. Leonardo's combinatorial model

CONCLUSION

Due to a broad scenario and interest in research involving forms of generalization of linear sequences and recurring, we have the combinatorial approach of Fibonacci and Lucas sequences as a basic contribution to this research.

On the other hand, the combinatorial approach is presented as an important aspect in research advances involving sequences. In this way, through the notion of board, we have the introduction of the combinatorial study of the sequence of Leonardo, allowing a study of these numbers in the combinatorial scenario of recurring sequences.

In the preceding sections, we explored basic problems in Combinatorics. Surprisingly, these problems led to connections with the Fibonacci Sequence. In summary, the problems and approaches discussed in the previous sections belong to the field of Pure Mathematics known as "Combinatorics," often referred to as Finite Mathematics because it focuses on studying finite objects. However, it's essential to note that there are infinitely many finite objects, and at times, it's beneficial to reason about all members of an infinite collection.

For the area of education, it is noted that by visualizing the terms of Leonardo's sequence, it is possible to advance the mathematical content and improve the Mathematics teaching process for the History of Mathematics area. Thus, the teacher will have a new possibility of approaching the content in the classroom, allowing a didactic transposition with the combinatorial interpretation of the sequence under study.

Given the presentation of the combinatorial approach of Leonardo's sequence, it can be noted that the objective of the work was achieved, carrying out a study of these numbers and allowing an improvement in the Mathematics teaching process.

This perspective on Combinatorics not only enhances the understanding of mathematics for teachers from a basic level but also illustrates its relevance in contemporary research related to Fibonacci numbers and Leonardo numbers.

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