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Students' Mathematical Communication: The Effectiveness of Somatic, Auditory, Visual, Intellectual Learning and Problem-Based Learning Model on Number Pattern

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Abstract

This study aimed to analyze the effectiveness of the Somatic, Auditory, Visual, Intellectual (SAVI) learning and Problem-Based Learning (PBL) models in improving students' mathematical communication ability compared to direct learning. A quasi-experimental design was employed, with the SAVI learning model implemented in experimental class 1, PBL in experimental class II, and direct learning in the control class. The sample was selected using cluster random sampling, and data were collected through observations, interviews, and tests. Hypothesis testing, including proportion tests, one-way ANOVA, and Scheffe tests, revealed the following results: (1) students taught with the SAVI learning model demonstrated significant improvement in mathematical communication ability; (2) students instructed using the PBL model also exhibited substantial enhancement in mathematical communication ability; (3) notable differences were observed among the SAVI learning, PBL, and direct learning models, highlighting the effectiveness of both the SAVI learning and PBL models in enhancing mathematical communication ability. Moreover, the PBL model outperformed direct learning, while the SAVI learning model showed greater effectiveness than direct learning. Overall, this study provides evidence supporting the efficacy of the SAVI learning and PBL models in fostering students' mathematical communication skills.

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INTRODUCTION

Mathematics is a science that has an important contribution to life. Hasratuddin (2015) said mathematics is also referred to as human life because mathematics is a product of human intellectual thinking. According to Hudojo (2005) mathematics is a tool to improve the way of thinking, so mathematics is indispensable both for daily life and the advancement of science and technology which makes mathematics need to be provided for every student since elementary education. Therefore, mathematics is an important science that needs to be taught in schools.

Permendikbud Number 58 of 2014 related to the 2013 Curriculum of junior high schools/Tsanawiyah formulates the purpose of learning mathematics so that learners have the ability to communicate opinions, and reasoning and are able to compile mathematical evidence using complete sentences, symbols, tables, diagrams, or other media to clarify circumstances or problems. To achieve these mathematics learning objectives, NCTM (2000) mentions five standards of ability that learners must have in learning mathematics, which are mathematical communication; mathematical reasoning; solving mathematical problems; mathematical connections; Positive attitude to mathematics. So in the mathematics learning process, it is hoped that teachers can come up with these five abilities. The abilities that must be developed in learning mathematics are mathematical communication ability.

Mathematical communication ability in mathematics learning really needs to be developed (Purnama, 2016). Baroody in (Ansari, 2016) states two important reasons for the need to develop mathematical communication. First, mathematics is a langue, meaning mathematics is not only a thinking aid, but mathematics is also a tool for solving problems and for communicating various ideas. Second, mathematics learning is a social activity, meaning mathematics is a means of communication between learners that can achieve mathematical understanding. Asikin in Kumalaretna and Mulyono (2017) also revealed the importance of meaningful mathematical communication as a tool to explore mathematical ideas and help learners' ability to see various interrelationships of mathematical material, a tool to measure the growth of understanding and reflect on mathematical understanding in learners. In addition, mathematical communication abilities are also used as a tool to organize and consolidate learners' mathematical thinking as well as a tool for constructing mathematical knowledge, developing problem-solving, improving reasoning, cultivating self-confidence, and improving social ability.

However, based on the results of an interview with a mathematics teacher at SMP Negeri 13 Magelang, it is stated that learners' mathematical communication abilities are still not optimal. Due to the lack of learning motivation and students' ability to solve story questions and questions in the form of abstract problems. Students consider mathematics to be a difficult subject because of the many calculations and formulas for solving problems in mathematics. This is also evidenced by the results of the initial test of the mathematical communication ability of class VIII learners with an average score of 48.37. This means that learners' communication abilities are still relatively low. According to Awaliyah, Soedjoko, and Isnarto (2016) one of the efforts to improve the learning process is to choose the right and innovative learning model in mathematics learning.

One of the activity-based learning models that are interactive, fun, and motivating students is the SAVI (Somatic, Auditory, Visual and Intellectual) learning model. Optimal learning is when students learn a little about concepts by looking at presentations (visual), but students can learn more when they can do something (somatic), talk, or discuss what they are learning (auditory), and think and draw conclusions or information that can be applied in solving problems (intellectual) (Juwina & Amalia, 2020). This is reinforced by research conducted by Magfiroh, Baiduru, and Ummah (2017) results were obtained that mathematics learning with the SAVI learning model can improve the communication ability of students.

Permendikbud Number 34 of 2018 concerning Standards for the Primary and Secondary Education Process said that PBL is one of the learning models recommended in the 2013 Curriculum in addition to Discovery Learning (DL) and Project Based Learning (PjBL). The advantage of the PBL model is to suppress student activity by using real problems as something that learners learn to get important concepts so that they can communicate in mathematics learning (Kurnila et al., 2022). PBL model makes learners more active so that it can improve Learners' mathematical communication ability (Iskandar, Ermiana, & Rosyidah, 2021). This is strengthened by research conducted by Susanti, Juandi, and Tamur (2020) it was found that mathematics learning with the PBL model affected improving learners' mathematical communication ability.

The purpose of this study is, (1) to analyze the level of mathematical communication of Learners taught with the SAVI learning model to achieve learning goals, (2) to analyze the level of mathematical communication ability of learners who acquire a PBL model to achieve learning goals, and (3) analyze the level of mathematical communication ability of learners who received the SAVI learning and PBL model are better than the direct learning model.

RESEARCH METHODS

This research is a quantitative study with quasi-experimental methods, while the design used is the nonequivalent posttest-only control group design. The population of this study was all class VIII learners at SMP N 13 Magelang. The sampling technique used Cluster Random Sampling.

The selected samples were class VIII E as experimental class 1 which was treated by being taught using the SAVI learning model, class VIII H as experimental class 2 which was treated using a PBL model, and class VIII G as a control class using a direct learning model. The data collection techniques used are observation, interviews, and tests of students' mathematical communication skills. This test consists of 5 written test questions in the form of description questions with the following scoring guidelines (Table 1).

Before conducting research, an initial test of students' mathematical communication skills is first carried out for preliminary data analysis. Preliminary data analysis performed included normality tests, homogeneity tests, and mean similarity tests. The results of the analysis that has been carried out, shows the results that the experimental class and the control class come from a normally distributed population, have the same variance and have the same average. Then the final data is obtained after the treatment through tests. The final data analysis carried out includes normality tests, homogeneity tests, due diligence, ANOVA one-way tests, and post-ANOVA advanced tests.

Table 1. Mathematical Communication Ability Test Scoring Guidelines

No	Indicators	Score	Valuation
1	Explain the idea or solution of a mathematical problem in the form of a drawing, diagram, or table.	0	No answer
		1	Students create patterns or tables but they are not quite right.
		2	Students create patterns or tables that are relevant to the problem, complete, and precise.
2	Explain the idea or solution of a problem or image using your own language.	0	No answer
		1	Only a few of the students' statements are true
		2	The student's explanation is mathematically reasonable and correct, even if it is not logically composed or there is a slight language error
		3	The student's explanation is mathematically reasonable and clear and logically arranged.
3	Expressing everyday problems or events in a mathematical model.	0	No answer
		1	Students write down the concept of mathematical formulas from the given problem correctly but do not carry out the calculation process.
		2	Students incorrectly write down the concepts of mathematical formulas used and do incorrect calculations.
		3	Students write down the concept of mathematical formulas from the given problem correctly but are wrong in doing the calculations.
		4	Students write down the concept of mathematical formulas from the given problem correctly
		5	Students write down the concept of mathematical formulas correctly and can do calculations correctly and precisely and write down the conclusions at the end of the answers correctly and correctly

RESULTS AND DISCUSSION

Pre-Requisite Test

Before the study was carried out, preliminary data analysis was carried out with normality tests (in Table 2), homogeneity tests (in Table 3), and average similarity tests (in Table 4). The data used are the initial test scores of the mathematical communication ability of class VIII learners.

Table 2. Summary of Normality Test Results

Class	L_{obs}	L_{critic}	Decision	Conclusion
Experimental 1	0.1465	0.161	$L_{obs}<L_{critic}$	H_0 accepted
Experimental 2	0.1167	0.161	$L_{obs}<L_{critic}$	H_0 accepted
Control	0.1032	0.161	$L_{obs}<L_{critic}$	H_0 accepted

Based on Table 2, it can be seen that $L_{obs}<L_{critic}$ then H_0 is accepted. It means that the experimental class 1, experimental class 2, and control class are normally distributed.

Table 3. Summary of Homogeneity Test Results

χ^2_{obs}	χ^2_{critic}	Decision	Information
0.5346	5.9915	$\chi^2_{obs}< \chi^2_{critic}$	H_0 accepted

Based on Table 3, it can be seen that the value $\chi^2_{obs}< \chi^2_{critic}$ then H_0 is accepted. It means that the experimental class and the control class are homogeneous.

Table 4. ANOVA Summary of Test Results

Variant Source	*JK*	*dk*	*RK*	F_{obs}	F_{critic}
Between Groups	291.4667	2	145.7333	0.4554	3.1013
Within Groups	27838,13	87	319.9785		
Total	28129.6	89			

Based on Table 4, it can be seen that $F_{obs}=0.455<3.101=F_{critic}$. The result H_0 is accepted, it means that the experimental class and the control class have the same average ability.

Final Data Analysis

The final data analysis will be carried out with normality (in Table 6), homogeneity (in Table 7), and effectiveness tests (proportion test, one-way ANOVA, and post-ANOVA follow-up).

Table 5. Summary of Final Data Results

Kelas	X_{max}	X_{min}	$\bar{X}$	S
Experimental 1	92	60	79.933	8.162
Experimental 2	94	70	82.533	7.700
Control	90	56	73.533	10.207

Based on Table 5, it can be seen that the final ability test results for experimental class 1 get a maximum score of 92, a maximum score of 94 for experimental class 2, and the control class gets a score of 90. The minimum score of 60, 70, and 56 respectively for experimental class 1, experimental 2, and the control class. While the average value of the experimental class 1 was 79.933, the average of the experimental class 2 was 82.533 and the control class was 73.533.

Table 6. Summary of Normality Test Results

Class	L_{obs}	L_{critic}	Decision	Conclusion
Experimental 1	0.0754	0.161	$L_{obs}<L_{critic}$	H_0 accepted
Experimental 2	0.1019	0.161	$L_{obs}<L_{critic}$	H_0 accepted
Control	0.1031	0.161	$L_{obs}<L_{critic}$	H_0 accepted

The conclusion of the Liliefors normality test is $L_{obs}<L_{critic}$ then H_0 is accepted. It means that the experimental class and the control class are normally distributed.

Table 7. Summary of Homogeneity Test Results

χ^2_{obs}	χ^2_{critic}	Decision	Conclusion
2,6661	5.9915	$\chi^2_{obs}< \chi^2_{critic}$	H_0 accepted

Based on Table 7, in the learning model category obtained values of $\chi^2_{obs}< \chi^2_{critic}$ then H_0 is accepted. It means that the experimental class and the control class are homogeneous.

Effectiveness Test

Proportion Test

The final data proportion test was used to determine the learning completion of the experimental class group that had achieved the learning objectives, which was 75% of the number of learners. Test the proportion in this study using the *Z*-test formula, and the results are summarized in Table 8.

Table 8. Final Data Proportion Test Results

Class	Z_{obs}	Z_{critic}	Decision	Conclusion
Experimental 1	2.319	1.64	$Z_{obs}>Z_{critic}$	H_0 accepted
Experimental 2	3.162	1.64	$Z_{obs}>Z_{critic}$	H_0 accepted

Table 8, it can be seen that the $Z_{obs}>Z_{critic}$ result is H_0 is accepted. It means that learners who get treatment with the SAVI learning and PBL model have achieved learning goals.

Mean Difference Test

The final data average difference test is used to determine the effectiveness of the SAVI learning, PBL, and direct learning model. This test uses one-way ANOVA.

Table 9. ANOVA Summary of Test Results

Variant Source	*JK*	*dk*	*RK*	F_{obs}	F_{critic}
Between Groups	1287.2	2	643.6	8.391	3.101
Within Groups	6672.8	87	76.609		
Total	7960	89			

Based on Table 9, it can be seen that F_{obs}=8.391>3.101=F_{critic}. As a result, H_0 was rejected, which means that the three samples had significant differences in the mathematical communication ability of class VIII learners in the matter of number patterns between the SAVI learning, PBL, and direct learning models.

ANOVA Post Follow-Up Test

The post-ANOVA follow-up test was used to determine the difference in the mean of each pair of rows, columns, and cells. Because this study only knew that the treatments studied did not have the same effect, but did not yet know which of the treatments was significantly different from the others, it was necessary to carry out a post-ANOVA follow-up test using the Scheffe method.

Table 10. Post-ANOVA Follow-up Test Results

H_0	F_{obs}	F_{critic}	Conclusion
$\mu_1=\mu_2$	1.32	5.42	H_0 accepted
$\mu_2=\mu_3$	15.84	5.42	H_0 accepted
$\mu_1=\mu_3$	8.01	5.42	H_0 accepted

Based on Table 10, the following results are obtained. (1) $F_{1-2}=1.32<5.42=F_{critic}$, then H_0 is accepted. This means that there is no difference in learners' mathematical communication ability between learners who received the SAVI learning and PBL model. (2) $F_{2-3}=15.84>5.42=F_{critic}$, then H_0 is rejected. This means that there are differences in mathematical communication ability given by the PBL and direct learning model. (3) $F_{1-3}=8.01>5.42=F_{critic}$, then H_0 is rejected. This means that there are differences in the ability of mathematical communication given the SAVI learning with the direct learning model.

Students' Mathematical Communication Using the SAVI Learning Model

Based on one party proportion test obtained $Z_{obs}=2.319>-1.64=Z_{critic}$. Because the value z_{obs} is not included in the critical area, it is H_0 accepted. This means that the mathematical communication ability of learners who are given the SAVI learning model achieves completion. Based on the final data results in experimental class 1 can be seen in Table 5 obtained the highest value is 92, the lowest value is 60 and the average obtained is 79.933.

In general, the achievement of the results obtained in experimental class 1 shows that the SAVI learning model has a positive influence based on the completeness of learning post-test scores based on rubrics and indicators of mathematical communication ability. As already explained, the SAVI learning model is a learning model that consists of four elements that must be involved when learning. The four elements are somatic, auditory, visual, and intellectual elements that make learners actively move and think during learning. The visual element in the SAVI learning model is that the teacher invites learners in groups to observe, read and collect appropriate information to solve problems in worksheet. Where it is known that visual learning is good if learners can see examples from the real world, diagrams, and idea maps and read. The visual element in this research is actively learning to solve worksheet which contains contextual problems and images and is active in collecting information.

On the auditory element or the element that continuously captures and stores the information heard. In this study, the auditory element was found in the material delivery activities by the teacher, learners were invited to listen to the teacher's explanation and pay attention to the explanation of their friends, and exchange ideas when discussing solving problems in worksheet. This is in line with the opinion of Ayundhita and Soedjoko (2014) that group discussion interaction has an important role in a problem and then trying to solve the problem in groups.

In the intellectual element, which means learning by solving problems and reflecting. For example, learning by solving problems that exist in worksheet. Where through the intellectual element learners think and analyze problems and then discuss them with their group friends so as to produce answers in worksheet.

In somatic elements, the teacher gives freedom to learners to stimulate the mind and body in the classroom so as to create a physically active learning atmosphere by inviting learners in groups to complete the worksheet that has been shared. After solving the problem at worksheet, learners presented the results of their group work in front of the class. In this study, there are difficulties in developing learning on somatic elements, so the learning activities of these somatic elements have not been maximized. This is because somatic element learning activities only rely on solving problems in worksheet and presenting the results of group work. This resulted in only some learners actively moving during learning.

The results of this study support the results of a study conducted by Siregar (2018) which states that the SAVI learning model is effective in improving learners' mathematical communication ability. The results of this study are also supported by the findings of Minarni (2019) who states that learners' communication ability in Indonesia still has the opportunity to improve if the learning model provided is accompanied by student-centered learning such as the SAVI learning model, it is even better if the teacher applies learning media. Mathematics learning media can help demonstrate or visualize mathematical concepts as well as tools for constructing mathematical concepts.

Students' Mathematical Communication Using the PBL Model

Based on one party proportion test obtained Z_{obs}=3.16>-1.64=Z_{critic}. Because z_{obs} are not included in the critical area, it is H_0 accepted. The mathematical communication ability of learners who obtained the PBL model achieved completion. Based on the post-test results in experimental class 2 can be seen in Table 5, the highest value is 94, the lowest value is 70 and the average is 82.533. The achievement of learning outcomes in experimental class 2 with the PBL model is not much different from the achievement of learning outcomes of experimental class 1 learner. This is because learners who follow the PBL model can involve in mathematical communication ability in learning through the PBL model learning stage.

In the first stage which is the orientation of learners to the problem, the teacher presents contextual problems that correspond to the material to be studied and motivates learners to be actively involved in solving a given problem. The second stage is organizing learners to learn, which is to divide learners into small groups, helping learners define problem-related learning tasks. The purpose of forming this group is so that learners are trained to learn in groups, discuss and exchange ideas in solving each problem in worksheet, and help provide understanding if there are group friends who do not understand the material. In addition, discussions in small groups result in learners giving questions that can train learners' mathematical communication ability when answering them. This is in accordance with the opinion of Mitasari and Prasetijo (2016) state that learners learning activities in small groups provide opportunities for learners to carry out mathematical communication through a number of questions focused on the nature of the problem, building previous knowledge with new knowledge and using the right strategies in solving problems.

The third stage guides individual and group investigations, where teachers encourage learners to gather as much information as possible and share their own ideas to answer the problems contained in the worksheet. The fourth stage is to develop and present results. At this stage, each group is asked to convey the results of their discussion, at the time of the presentation learners convey ideas, arguments, and solutions to the given problems. For groups that do not present, they are asked to argue if there is something that is not yet understood or if there are different answers. This is in line with the opinion of Ayundhita and Soedjoko (2014) that learning using presentations is able to build learners to express student ideas derived from what is obtained when solving problems. The interaction in the Q&A session resulted in learners being able to express their ideas as freely as possible during the presentation process. In the last stage, namely analyzing and evaluating, the teacher provides reinforcement if there are difficulties in drawing conclusions from the results of the presentation and question and answer. And check learners' understanding of the learning that has taken place by giving exercises to learners.

The results of this study are in accordance with research conducted by Binjai (2019) which resulted in the conclusion that mathematics learning with the PBL model has a significant effect on learners' mathematical communication ability. In addition, the results of this study are also in line with the findings of Perwitasari and Surya (2021) which state that improving learners' mathematical communication ability in the mathematics learning process cannot be separated from the selection of learning models. The PBL model is a classroom strategy that organizes mathematics learning with everyday problems so as to provide more opportunities for learners to develop critical thinking ability, present ideas in their own language, as well as communicate mathematically with peers. Based on the syntax of the PBL model, the learning model focuses on learner activity and contextual problems to be given to learners. So that learners' understanding of the material provided becomes more optimal and results in the development of learners' mathematical communication ability.

Mathematical Communication: SAVI, PBL, and Direct Learning Model

Based on the calculation of the ANOVA one-way test, it was obtained that $F_{obs}=8.391>3.101=F_{critic}$. Because the value F_{obs} is included in the critical area, it is H_0 rejected. This means there are differences in the level of effectiveness of mathematical communication ability between learners who obtain the SAVI learning, PBL, and direct learning models in class VIII number pattern material.

Based on the post-ANOVA follow-up test, a decision was obtained that experimental class 1 and experimental 2, it was obtained that the test decision were H_0 accepted. This means that SAVI learning and PBL models are both effective in improving learners' mathematical communication ability. The SAVI learning and PBL models are learning models that both focus on student activities during the learning process. Both have cooperative elements where learners discuss and exchange ideas in solving given contextual problems with other learners. This is in accordance with the constructivism theory stated by Vygotsky (in Pohan, 2020), which means the learning process will be efficient and effective if learners learn cooperatively with peers in a supportive atmosphere and environment, under the supervision of someone more capable.

Then in experimental class 2 and control, it was obtained that the test decision was H_0 rejected. This means that there is a difference between PBL and direct learning model against mathematical communication ability. In this study, The average score of experimental class 2 is 82.53, while in the control class, the average is 73.53. Based on these averages, it is concluded that the PBL model is more effective than the direct learning model for learners' mathematical communication ability. Learning in experimental class 2 with the PBL model is carried out in groups and started with contextual problems so that learners construct their own knowledge. This is in line with Ausubel's meaningful learning theory which utilizes contextual problems, guiding learners to use their own way of solving these problems until they find a concept. So that learning mathematics is more meaningful.

Then in experimental class 1 and control, it was obtained that the test decision was H_0 rejected. This means that there are differences in the effectiveness of the SAVI learning model and the direct learning model on mathematical communication ability. The average value of experimental class 1 was 79.93, while the average control class is 73.53. Based on these averages, it can be concluded that the SAVI learning model is more effective than the direct learning model against the mathematical communication ability of learners. The learning process in experimental class 1 with the SAVI learning model is carried out in groups and given practice questions related to contextual problems, so that learners actively discuss solving a given problem, and presented them in front of others. Learners who are not presenting also actively listen to their friends during the presentation and actively ask questions if there are things they still don't understand. This is in accordance with the theory of learning according to Piaget, where learners are encouraged to actively participate in building understanding based on experience and interactions that occur.

Learning in control classes, learners tend to passively listen to what the teacher explains. The teacher becomes the center during the learning process by providing material concepts and providing examples of questions and their solutions. Learners have a chance to be active when the teacher gives questions in a textbook or blackboard to work on. Occasionally teacher gives an opportunity for learners to write the results of their answers on the blackboard.

CONCLUSION

Based on the discussion above, it can be concluded that: (1) The results of the mathematical communication ability test of students who obtained the SAVI learning model have reached completion. (2) The results of the mathematical communication ability test of students who obtained the PBL model have reached completion. (3) There is a difference between the results of the student's mathematical communication ability test between students who obtained the

SAVI learning, PBL model, and the direct learning model. The third hypothesis is outlined as follows. (a) The SAVI learning and PBL models are equally effective against students' mathematical communication skills. (b) The PBL model is more effective than the direct learning model of students' mathematical communication skills. (c) The SAVI learning model is more effective than the direct learning model of students' mathematical communication skills.

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Influence a Realistic Mathematics Education Approach and Motivation on Students' Mathematical Reasoning Ability

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Mathematical Reasoning Ability; Motivation to Learn; Realistic Math Education Approach.

Abstract

The purpose of the research was to provide knowledge of the effect of the Realistic Mathematics Education (RME) approach and motivation on students' mathematical reasoning abilities. This quasi-experimental study employed a pretest-posttest control group design, focusing on fifth-grade students at SDN Lulut 05, Klapanunggal District. Class VA was selected as the experimental group, while class VB served as the control group using purposive sampling. Data analysis involved the *N*-gain formula and *t*-test. The results shown: (1) There is a difference in the increase in students' mathematical reasoning ability after being given the RME approach indicated by the N-gain value of 55.5 in the medium category and for the experimental class 36.8 in the low category, then the control class in the *t*-test (independent) so that the Sig value is obtained. $0.001<0.005$, as for deciding in the independent sample *t*-test. It is concluded that there is a significant difference between the average score mathematical reasoning ability by students treated with the RME approach and the Conventional approach. (2) There is an influence of the RME approach on students' mathematical reasoning abilities with an effect size score of 1.44 which is in the high category.

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INTRODUCTION

Educational goals can optimally be realized by presenting quality learning, for this there are several points to pay attention to, including planning student-centered learning, especially related to the skills or competence of educators (Charoline et al., 2020). Educators are facilitators in learning, meaning that educators provide direction and facilities for students to develop their potential, to become effective educators according to (Stronge, 2007) "effective educators demonstrate high expectations for students and select strategies to propel the students' learning. Beyond planning and preparation of materials, effective organizing for instruction also involves the development of a conscious orientation toward teaching and learning as the central focus of classroom activity. Teaching and learning as a focus must be consistently communicated to students in the classroom and to observers that effective educators display high expectations for students and choose strategies to encourage student learning. Beyond planning and preparing materials, effective organizing for teaching also involves developing a conscious orientation towards teaching and learning as the primary focus of classroom activities. Teaching and learning as a focus must be communicated consistently to students in class and to observers.

According to Freudenthal, mathematics is not a body of mathematical knowledge, but the activity of solving problems and looking for problems, and, more generally, the activity of organizing material from reality or mathematical material - which is called "mathematizing". he clarified what mathematics was: "mathematics without mathematization does not exist". For this reason, educators need to find out the context, create appropriate contexts support students to construct mathematical knowledge. some other suggestions for educators to find and create contexts for learning mathematics, namely: the history of mathematics in context; context in real life (life of elementary school students: games, shopping, saving and using money, movies,...; social issues: traffic, weather forecast, lottery, ...); integrated education (mathematics Physics, Chemistry, Information Technology, etc.) (Trung, Thao, & Trung, 2019).

Another opinion argues that realistic mathematics learning is the use of reality as well as the environment so that students can understand it so that the mathematics learning process can run optimally, and achieve the goals of mathematics education better than in the past and approaches in learning use contexts that can make students describe their learning and real in the minds of students. This is in accordance with the stages of development of elementary school students who are in the concrete operational stage, especially elementary school students aged 7-12 years (Herzamzam & Rahmad, 2020; Rimadona, Fitriani, & Robandy, 2018; Suprayogo, Sutrisno, & Supandi, 2019). The RME model focuses on learning mathematics which focuses on the daily lives of students (contextual) which presents things that are essentially real to be taught to students using the RME approach and its principles

Mathematics learning is expected to be based on contextual problems, so that students understand more about mathematical material, and it will not be difficult for students to understand abstract mathematical material. Gusnarsi, Utami, and Wahyuni (2017), Putri, Isrok'atun, and Kurnia (2017), Hasan, Pomalato, and Uno (2020), Ariati and Juandi, (2022) the role of the educator must be able to carry out the RME approach. According to Trung et al. (2019) several important principles of teaching Mathematics based on RME as follow. The first, activity Principle. The learner as an active subject in the teaching process whose activity is a key factor for the outcome of this process. Therefore, the best way to learn Mathematics is by solving mathematical problems. The second is the reality principle. Requires students to apply mathematical knowledge by solving practical problems and mathematics education must start from meaningful practical situations with learners in order to provide opportunities to work with these meanings into the structure of mathematics in the minds of students. The third is the principle of levels. It is emphasizing cognitive development with various levels of learning mathematics: from non-mathematical contexts that involve knowledge, with symbols, diagrams, to the content of pure mathematical knowledge. This approach to learning is important as a means between informal experience, the mathematical context involved, and pure mathematical knowledge. The approach here can be understood as mathematical modeling.

Fourth, the principle of interweaving: Students are placed in various situations where they can perform various types of interrelated tasks (reasoning, calculating, statistics, performing algorithms, etc.), using many Mathematical knowledge and tools from different disciplines, even other sciences. Fifth, principle of Interactivity: encourage interpersonal and group activity to create opportunities for individuals to share skills, strategies, inventions, ideas, etc. with other students. In return, they can benefit from others for cognitive and personal development. Sixth, the principle of guidance can be described as a guided reinvention process in mathematics instruction. In particular, educators need to design scenarios or situations (or contexts) that are potentially rich in activities, the implementation of which can create meaningful cognitive leaps for students.

So important is the role of educators in presenting lessons, especially mathematics, it is not uncommon for some students to dislike mathematics because thinking about mathematics is a subject that has been considered difficult so far. Based on this, effective educators will be able to increase student motivation and present lessons that arouse the interests and needs of students, with high motivation in a lesson, the hope for the realization of learning will be achieved optimally, this is in line with the opinion of Suprayogo et al. (2019) that students who have high learning motivation, have enthusiasm and have high enthusiasm in learning and do not give up easily solving problems, the higher the learning motivation of students, the better their learning achievement. Students who have high learning motivation will easily understand the questions and determine the direction of solving the problems being worked on.

To turn on learning motivation, a learning approach is needed so that it answers the characteristics of abstract mathematics through an approach that is based on real or realistic, this approach is known as the realistic math education (RME) which is a solution that is considered appropriate in answering these characteristics of mathematics, because realistic mathematics which allows learning emphasizes all aspects of the potential of students to develop, here the role of the educator again becomes an important thing in generating motivation through the approach realistic math education (RME). Another is how the approach is expected to be able to turn on motivation and improve mathematical reasoning abilities.

Motivation as a mental drive can move and give direction to human behavior, one of which is learning behavior. In motivation there is a desire that makes active, moving, channeling, and, giving direction to the attitudes and behavior of a person learning (Dimyati & Mudjiono, 2006), based on this, educators must be able to build student motivation to enjoy and like a subject (Rahman & Mirati, 2019) is in line with their opinion. According to Munawaroh, Santosa, and Wahyuningrum (2020) motivation has a large influence on teaching and learning activities carried out by educators. learning and in education. To be able to bring out the motivation of students in learning,

In general, motivation is an encouragement to change the energy within yourself into the form of real activity in order to achieve the goals you want to achieve. The function of motivation itself, including: (1) giving encouragement so that behavior or actions arise, if motivation is not visible then an action does not appear as an example of learning; (2) as a direction, the intention is to move actions towards achieving the desired goal; (3) as a mover. Motivation has a role like a machine, meaning that the amount of motivation can determine the speed in a job (Octavia, 2021).

Reasoning ability is one of the important mathematical abilities in mathematics, according to what is stated in the book (Van de Walle, Karp, & Bay-Williams, 2014) namely "Reasoning and Proof Standard Instructional programs from prekindergarten through grade 12 should enable all students to Recognize reasoning and proof as fundamental aspects of mathematics, Make and investigate mathematical conjectures, Develop and evaluate mathematical arguments and proofs, Select and use various types of reasoning and methods of proof"that the Reasoning and Proofing Standards Instructional programs from kindergarten to grade 12 should enable all learners to recognize reasoning and proof as basic aspects of mathematics, make and investigate mathematical conjectures, develop and evaluate mathematical arguments and proofs, select and use different types of reasoning and the method of proof.

According to Nababan (2020) that mathematical reasoning can be classified into two types, namely inductive reasoning and deductive reasoning. Inductive reasoning is based on cases or examples to observe and draw general conclusions. This reasoning makes it easier to map the problem so that it can be used for other similar problems. Other forms of inductive reasoning include: analogy conclusions, generalizations, evaluation or assessment of answers and the process of solving and forming hypotheses. Inductive reasoning is classified into low or high mathematical reasoning depending on the complexity of the situation. Deductive reasoning is a reasoning process derived from general knowledge of principles or experience to conclusions about something specific (Ramdani, 2012). Deductive considerations include: Performing arithmetic operations, drawing logical conclusions, providing explanations about patterns, facts, properties, relationships or patterns.

Improving students' logical abilities during the learning process is the main aspect in the framework of successful learning. The higher the students' reasoning, the faster learning indicators can be achieved. Every math lesson, educators must always practice and expand mathematical thinking. Mathematical thinking is important both when creating proving or testing programs, but also when educators think with artificial intelligence systems (Nababan, 2020). In addition, according to Afinadhita and Abadi (2022) mathematical reasoning can make students think logically, critically, effectively and efficiently. This way of thinking can train students to be able to solve problem-based questions.

RESEARCH METHODS

The research was carried out at SDN Lulut 05, Klapanunggal District, Bogor Regency for fifth grade students in the even semester of the 2023-2024 academic year. Descriptive research pattern, with a quantitative approach. the research object and these variables must be defined in the operational form of each variable. This research is a quasi-experimental research that aims to see the effect of the RME approach, motivation and reasoning abilities on mathematics learning outcomes. The research population consisted of all fifth grade students at SDN Lulut 05, consisting of 63 students, with a composition of 31 male students and 32 female students. Sampling technique using purposive sampling; based on needs if it is suitable as a data source or sample with certain aspects that are believed to be able to produce maximum data. This sampling was based on consideration of the homogeneity of students which was also supported by statements from school principals, educators, and school staff. which said that the two classes used as samples had the ability to use them as research samples. In this study, the sample used was 31 students in the VA class which was the control class and VB class which consisted of 32 students at SDN Lulut 05 which was the experimental class.

RESULT AND DISCUSSIONS

The results of the research that will be presented are in the form of students' mathematical reasoning abilities on the subject of fractions using RME approach and students' learning motivation towards learning by using a RME learning model for the results of students' mathematical reasoning ability tests in this study were obtained from the results of students' pretest and posttest answers in both the experimental class and the control class. The value calculation data pretest and posttest students in the experimental class and control class can be seen in Table 1.

Tabel 1. Recapitulation of the *N*-Gain Test of Mathematical Reasoning abilities Experiment Class and Control Class Students

Class	Average Value		Gain	Category
	Pretest	Posttest		
Experiment	67.5	85.3	55.5	Currently
Control	63.7	77.7	36.8	Low

From the results of the pretest and posttest that have been carried out in the experimental class and the control class, the results show that mathematics learning using a RME approach on fractional teaching materials can improve students' mathematical reasoning abilities. This can be seen through the results of students' mathematical reasoning ability tests in the form of pretest and posttest which includes four indicators of mathematical reasoning ability including (1) the ability to present mathematical statements orally, in writing, pictures and diagrams, (2) the ability to make conjectures, (3) the ability to manipulate mathematics and (4) draw conclusions from statements.

Table 2 Data Normality Test Recapitulation

Data	Kolmogorov Smirnov			Shafiro Wilk		
	Statistic	*df*	*Sig.*	Statistic	*df*	*Sig.*
Pretest Eksperimen (RME)	.127	32	.200	.934	32	.052
Posttest Eksperimen (RME)	.140	32	.112	.934	32	.088
Pretest Control (Conventional)	.146	31	.093	.945	31	.113
Posttest Control (Conventional)	.209	31	.001	.843	31	.000

In the experimental class given treatment using a RME learning model, obtained result spretestie 67.5 increased to 85.3 so that an n-gain value of 5.55 was obtained in the medium category. As for the control class that uses a learning model conventional, obtained result spretestie 63.7 increased to 77.7 so that an n-gain value of 36.8 was obtained in the low category. This shows an increase between the experimental class and the control class is different. Therefore, homogeneity tests and normality tests were carried out as prerequisites for conducting the *t*-test. The results of the data normality test can be seen in Table 2.

Based on the output results above, it is known that the significance of Shapiro Wilk for the Pretest and Posttest variables is greater than 0.05, so it can be concluded that the variables are normally distributed. Then the *t*-test was carried out and the results of the *t*-test calculations can be seen in Table 3.

Table 3 The Calculation Results

	Levene's Test analysis		*t*-test analysis		
	F	*Sig*	*t*	*df*	*Sig*. (2-tailed)
Equal variances assumed	.360	.551	3.642	61	.001

It can be seen from the table that the *Sig*. Levene's Test for equality of Variances of 0.551>0.05 so that it means that the data variant is between group A and group B are homogeneous or the same. And based on the Independent Sample Test table in section Equal Variances assumed known value of Sig. 0.001<0.005, then as the basis for taking decision in the independent sample *t*-test it can be concluded that H_0 is rejected and Ha is accepted, so there is a significant difference between the average learning outcomes of students with the treatment by RME approach and the conventional approach. The results of the research are fully based on the learning theory put forward by Piaget that the basis for developing one's knowledge is the ongoing adjustment of one's mind to the surrounding reality and it can be concluded that learning uses the RME is able to improve students' mathematical reasoning abilities. From the results of calculations using effect size gain value effect size of 1.44 this shows the influence of the RME approach on students' mathematical reasoning abilities in Fractional language class V with high criteria.

The results of the last discussion are the learning motivation of all students towards learning using the RME approach through the results of the student learning motivation questionnaire which consists of 10 statements and students answer with what they feel, especially after receiving the learning process. The results of the student learning motivation questionnaire can be seen in Table 4.

Table 4. Recapitulation of Student Learning Motivation Questionnaire

Total	Average	Category
1320	45.20	High

The results of the student learning motivation questionnaire show that the RME approach has an influence to motivate students' learning. This can be seen through the results of the overall student learning motivation questionnaire of 45.20. If we relate it to the motivational questionnaire assessment criteria that 45.20 lies between $40.81 < X \leq 50.40$ this shows the learning motivation of students who are in the high category and it can be said that the RME learning approach has an influence on students' learning motivation.

CONCLUSION

Based on the results of the research that has been carried out, it can be concluded that: 1) there are differences in the increase in students' mathematical reasoning abilities after using the RME learning model in learning mathematics. This is evidenced by the N-gain value obtained, which is equal to 55.5 with the moderate category in the experimental class and 36.8 with the low category in the control class and the t-test (independent) obtained Sig values. 0.001<0.005, then referring to the basis of decision making in the independent test sample T Test it can be concluded that Ho is rejected and Ha is accepted, so there is a significant difference between the average learning outcomes of students treated with the RME approach with the Conventional; 2) There is the influence of the RME learning model on students' mathematical reasoning abilities which can be proven by the acquisition of the Effect Size score, which is equal to 1.44 in the high category; 3) There is the influence of the RME learning model on the motivation of students which can be proven by the acquisition of an average score of learning motivation of students as a whole which is equal to 45.20 in the high category.

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The Influence of Connected Mathematics Project (CMP) Learning Models on Student's Mathematical Communication Ability in View of Self Esteem

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Abstract

This study examines the effectiveness of the Connected Mathematics Project (CMP) learning model in improving students' mathematical communication skills at MA Madarijul Huda Pati. Because of 46.15% of students demonstrating very low skills and 23.07% falling into the low category, the study aims to: (1) analyze the impact of CMP on mathematical communication skills, (2) compare skills among students with high, moderate, and low self esteem, and (3) explore the interaction between CMP and self esteem. Employing a quantitative quasi-experimental approach, the study involves class XI students selected through cluster random sampling. Results indicate a significant improvement in mathematical communication skills through CMP. Students with high self esteem outperform those with moderate self esteem, while those with moderate self esteem fare better than those with low self esteem. Additionally, students with high self esteem exhibit superior mathematical communication skills compared to students with low self esteem. Notably, an interaction between the learning model and self esteem was observed, influencing mathematical communication skills.

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INTRODUCTION

Apart from playing an important role in everyday life, mathematics is also a compulsory subject studied at various levels of education. The ability to communicate mathematically is a crucial component that students must possess when learning mathematics. In the classroom setting, mathematical communication can occur between teachers and students, between textbooks and students, and among students themselves (Purnama & Alfriansyah, 2016). The National Council of Teachers of Mathematics (NCTM) (2000) suggests that one of the objectives of learning mathematics is to develop mathematical communication skills. Additionally, Regulation of the Indonesian Minister of National Education Number

22 of 2006 concerning Content Standards states that one of the goals of learning mathematics is for students to be able to communicate ideas using symbols, tables, pictures, or other media to clarify situations or problems.

The CMP learning model is an instructional approach that emphasizes learning tasks to actively engage students and promote discussions, thereby enhancing their mathematical communication skills. However, based on observations at MA Madarijul Huda Pati, teachers still predominantly employ direct instruction and do not utilize a variety of learning models. Consequently, students tend to be passive and do not actively participate in the learning process. A significant percentage of students at MA Madarijul Huda Pati have mathematical communication skills categorized as very low (46.15%) or low (23.07%). These statistics indicate that the desired goals of mathematics education have not been adequately achieved. The Program for International Student Assessment (PISA) survey conducted in 2018 revealed that Indonesia ranked 73 out of 78 participating countries, with an average score of 379, compared to the OECD average score of 489 (OECD, 2019). Furthermore, Fitria and Handayani (2020) found that students' communication skills in Indonesia are still categorized as low, while Yanti, Melati, and Zanty (2019) reported that students' communication skills, particularly in answering questions, are relatively low.

The success of mathematics education can be measured by students' mathematical communication skills. Yuniarti et al. (2018) conducted research showing a positive relationship between students' mathematical communication skills and their self esteem. Aspriyani (2020) also found that students with higher self esteem tend to have higher mathematical communication skills. As educators, teachers are expected to enhance self esteem as an effort to improve students' mathematical communication skills during instruction. Choosing the appropriate and innovative learning model in mathematics education is a fundamental necessity. According to Ziana and Ristontowi (2020), the CMP learning model is an effective approach for improving mathematical communication skills. By implementing the CMP learning model, it is hoped that a conducive learning environment can be created where students gain independence in expressing ideas, asking questions, and solving problems, thereby supporting the enhancement of their mathematical communication skills.

Based on the aforementioned background, the research questions are as follows: (1) How does the CMP learning model influence students' mathematical communication abilities? (2) Are there differences in mathematical communication skills among students with high, moderate, and low self esteem? (3) Is there an interaction between the CMP learning model and self esteem regarding mathematical communication skills? The objectives of this study are to (1) analyze the impact of the CMP learning model on students' mathematical communication skills, (2) compare mathematical communication skills among students with high, moderate, and low self esteem, and (3) examine the interaction between the CMP learning model and self esteem in relation to mathematical communication skills.

RESEARCH METHODS

This study used a quantitative approach with experimental research methods. The independent variables in this study were learning models and self esteem, while the dependent variable used was students' mathematical communication skills. The population used in this study were students of class XI MA Madarijul Huda Pati. The sampling technique used is cluster random sampling, namely group sampling, where the units selected are not individuals, but a group of individuals who are naturally together in one place. In this study, samples were taken using random samples with a lottery system so that each class had the same opportunity to be sampled in the study. The technique is to draw a roll of paper for a number of classes with the class number written on it, so that one experimental class and one control class are obtained. Data collection used instruments in the form of self esteem and mathematical communication ability test questions. The data analysis technique in this study is the two-way ANOVA test.

Connected Mathematics Project (CMP) Learning Model

The measurement scale in this study used a nominal scale with two categories, namely the CMP learning model in the experimental class and the direct learning model in the control class.

The CMP learning model is a learning model in which students are given the opportunity to develop and create their own knowledge by finding solutions to the given problems and then ending with discussions in class to ensure knowledge and obtain more effective and efficient solutions. The most basic difference between the CMP model and other models is that the CMP model has simple learning steps, as according to Lappan et al. (2002), the steps of the CMP learning model are launching problems, exploring, and summarizing. While the direct learning model is a simple learning model in which the teacher explains the subject matter directly to students and students listen. The steps of the direct learning model according to Slavin (2018), namely: informing learning objectives and lesson orientation to students, reviewing prerequisite knowledge and skills, conveying subject matter, implementing guidance, providing opportunities for students to practice, assessing performance and providing feedback back, as well as providing self-training.

Mathematical Communication Ability

Mathematical communication ability is an ability to express mathematical ideas or ideas in a structured manner with the aim of providing interpretation. The indicators of mathematical communication skills used in this study are based on Brenner (1998) which has been modified: (a) Problem solving tool, expressing problems into written mathematical ideas followed by writing complete and structured problem solving steps; (b) Alternative solution, providing an interpretation of the results of problem solving that has been obtained by writing conclusions. Use the results that have been obtained to solve other mathematical problems if needed.

In the problem solving tool indicator, students are expected to be able to write down mathematical ideas and steps to solve problems related to the problem of determining sigma notation and mathematical induction completely and accurately. Whereas in the alternative solution indicator, students are expected to be able to write conclusions based on the results of the completion previously obtained completely and correctly.

Data collection was based on the results of the pretest and posttest of mathematical communication skills in the form of a mathematics induction material description test which was implemented using the CMP learning model and direct learning.

Self Esteem

Self esteem is a self-assessment made by individuals towards themselves positively based on relationships with other people, so that their behavior reflects what the individual values and how individuals accept themselves. Students' self esteem is measured through a questionnaire based on Rosenberg (1965) Self Esteem Scale. RSES (Rosenberg Self Esteem Scale) consists of ten statement items. The scale used in this study consists of unfavourable items and favorable items. Favorable statements are statements that contain positive or supportive things about the attitude object. An unfavorable statement is a statement that contains negative things, namely not supporting the object of the attitude to be expressed.

RESULT AND DISCUSSION

The central tendency of the pretest posttest data of mathematical communication ability in the control class (direct learning model) and the experimental class (CMP learning model) is presented in Table 1.

Table 1. Central Tendency

Central Tendency		X_{min}	$\bar{X}$	M_e	M_o	X_{max}	SD
Control class'	Pretest	13.33	22.50	23.33	26.67	30.00	5.01
	Posttest	40.00	60.21	60.00	60.00	73.33	8.33
Experimental class	Pretest	16.67	22.50	26.67	33.33	4.49	4.49
	Posttest	50.00	73.12	76.67	76.67	90.00	10.47

The results of filling in the student self esteem questionnaire were processed and then classified into three self esteem groups namely high, moderate and low categories. In the experimental class there were 14 students with high self esteem, 14 students with moderate self esteem, and 3 students with low self esteem, while in the control class there were 9 students with high self esteem, 15 students with moderate self esteem, and 8 students with low self esteem.

Table 2. *t*-test Analysis

Statistics	Direct Learning Model	CMP Learning Model
Sum	720.00	796.67
Average	22.50	25.70
Standard Deviation	5.01	4.49
Variance	25.09	20.14
$S_{combined}$	28.07	
$\sqrt{\frac{n_1 + n_2}{n_1 \, x \, n_2}}$	0.25	
t_{count}	-0.45	
t_{table}	2.00	

In the initial data analysis, the pretest data for the control class and the experimental class are normally distributed and homogeneous. Based on the results of the *t*-test calculation, the pretest data average value of the experimental class' communication skills is comparable to the control class. The results of processing the *t*-test data are presented in Table 2.

The calculation of the t-test uses a significance level of 5%, where the t_{count}=-0.45 and t_{table}=2.00 are obtained. Based on the results of calculating the value of $t_{count}<t_{table}$, then is accepted, which means that the pretest average value of the experimental class' communication skills is comparable to the control class.

During the final data analysis, it was observed that the posttest data in both the control class and the experimental class exhibited normal distribution and homogeneity. Furthermore, the data for students with high, moderate, and low self esteem also followed a normal distribution and displayed homogeneity. The hypothesis testing involved conducting a two-way ANOVA test to compare the differences in data influenced by two independent variables, namely the learning model and self esteem. The results can be seen in Table 3.

Table 3. Summary of Two-Way ANOVA Test Analysis Results

Source	*JK*	*DK*	*RK*	F_{obs}	F_{α}	Test Decision
Learning Model (A)	868.08	1	868.08	11.72	4.01	H_{0A} is rejected
Self Esteem (B)	1262.02	2	631.01	8.52	3.16	H_{0B} is rejected
Interaction (AB)	670.38	2	335.19	4.52	3.16	H_{0AB} is rejected
Error	4223.16	57	74.09	-	-	-
Total	7023.64	62	-	-	-	-

Based on Table 3, the value $F_{obs(A)}$=11,72>$F_{\alpha(A)}$=4,01; $F_{obs(B)}$=8,52>$F_{\alpha(B)}$=3,16; and $F_{obs(AB)}$=4,52>$F_{\alpha(AB)}$=3,16. From the results of these calculations, a test decision can be taken, namely H_{0A} is rejected, H_{0B} is rejected, and H_{0AB} is rejected. Due to the rejection of the hypothesis, further tests were carried out using the Scheffe test.

The Scheffe test was carried out as a follow-up test of the two-way Anava which aims to find out which treatment pairs are significantly different due to the rejection of H_0. The follow-up test in this study used the mean comparison between rows to answer hypothesis 1, the mean comparison between columns to answer hypothesis 2, and the mean comparison between cells in the same column to answer hypothesis 3 on two-way ANOVA. The results of the mean test between rows to answer hypothesis 1 are presented in Table 4.

Table 4. Comparison of Means Between Rows

Learning Model	Self Esteem			Marginal Mean
	High	Moderat	Low	
CMP Learning	78.33	71.43	56.67	68.81
Direct Learning	61.85	60.22	58.33	60.14
Marginal Mean	70.09	65.83	57.50	-

Based on Table 4, the marginal mean value in answering hypothesis 1 for treatment using the CMP learning model is 68.81 and the marginal average for treatment using the direct learning model is 60.14, which means that 68.81>60,14.

This means that the mathematical communication skills of students who get learning using the CMP model are better than students who get learning using the direct model.

Furthermore, the mean comparison table between columns to answer hypothesis 2 is presented in Table 5.

Table 5. Comparison of Means Between Columns

Comparison	Computing	Critical Area	Test Decision
μ_1 vs μ_2	6.77	$\{F \mid F>6.32\}$	μ_1 is better μ_2
μ_2 vs μ_3	6.47	$\{F \mid F>6.32\}$	μ_2 is better μ_3
μ_1 vs μ_3	19.70	$\{F \mid F>6.32\}$	μ_1 is better μ_3

Note: μ_1=Students' mathematical communication skills with high self esteem; μ_2=Students' mathematical communication skills with moderate self esteem; μ_3=Students' mathematical communication skills with low self esteem.

Based on Table 5, the value $F_{1\text{-}2}$=6.77; $F_{2\text{-}3}$=6.47; dan $F_{1\text{-}3}$=19.70. The critical area in the mean comparison test between columns gets the value $\{F|F>6.32\}$ and in determining the test decision can be done by comparing F_{obs} with the critical area. Thus, there appears to be a significant difference in μ_1 and μ_2, μ_2 and μ_3, and μ_1 and μ_3. This means that students with high self esteem categories have better mathematical communication skills than students who have moderate self esteem categories, students with moderate self esteem categories have better mathematical communication skills than students who have low self esteem categories, and students with high self esteem category has better mathematical communication skills than students who have low self esteem category.

The results of average comparisons between cells in the same column to answer hypothesis 3 is also presented in Table 6.

Table 6. Comparison of Means Between Cells in the Same Column

Comparison	Computing	Critical Area	Test Decision
μ_{11} vs μ_{21}	20.09	$\{F \mid F>11.9\}$	μ_{11} is better μ_{21}
μ_{12} vs μ_{22}	12.27	$\{F \mid F>11.9\}$	μ_{12} is better μ_{22}
μ_{13} vs μ_{23}	0.08	$\{F \mid F>11.9\}$	μ_{13} is better μ_{23}

Note: μ_1=The CMP learning model and high self esteem towards mathematical communication skills; μ_{12}=The CMP learning model and moderate self esteem towards mathematical communication skills; μ_{13}=The CMP learning model and low self esteem towards mathematical communication skills; μ_{21}=The direct learning model and high self esteem towards mathematical communication skills; μ_{21}=The direct learning model and moderate self esteem towards mathematical communication skills; μ_{21}=The direct learning model and low self esteem towards mathematical communication skills.

Based on Table 5, the value $F_{11\text{-}21}$=20.09; $F_{12\text{-}22}$=12.27; and $F_{13\text{-}23}$=0.08. The critical area in the mean comparison between cells in the same column gets the value $\{F \mid F>11.9\}$. In determining the test decision can be done by comparing the F_{Obs} with the critical area. There are significant differences in μ_{11} and μ_{21}, and μ_{12} and μ_{22}. This means that Students with high, moderat, and low self esteem who get the CMP learning model have better mathematical communication skills than students with high, moderat, and low self esteem who get learning with the direct learning model.

Students' Mathematical Communication: CMP vs. Direct Learning Model

From the calculation results of the ANOVA test for two different treatment groups, the observed F-value ($F_{Obs(A)}$) is 11.72, which is greater than the critical F-value ($F_{\alpha(A)}$) of 4.01. Therefore, we can conclude that the null hypothesis H_{0A} is rejected, indicating that the learning model has an effect on students' mathematical communication skills. Based on the results of the mean comparison between groups, it is evident that the average score for the CMP learning model is 68.81, while the average score for the direct learning model is 60.14. This indicates that students who received the CMP learning model exhibited better mathematical communication abilities compared to those who received the direct learning model.

Factors that can cause students in classes taught using the CMP model to have better mathematical communication skills, because (1) student-centered learning, (2) learning is carried out in groups, so students can cooperate in solving the problems given, and (3) there are discussions with group members so that students can communicate both orally and in writing. In addition, the steps in the CMP learning model are also able to facilitate students to improve their mathematical communication skills as shown by the posttest results of mathematics.

Students who get learning using direct learning models are less enthusiastic about participating in the learning process. During the learning activities students only listen, listen, and record the material delivered by the teacher. Students tend to be passive and less enthusiastic about participating in learning activities. Relevant previous research, namely research conducted by Ziana and Ristontowi (2020) which states that the CMP learning model has an average result of mathematical communication skills that is better than conventional learning models. Apart from that, Isnani, Masykur, and Andriani's (2021) research also states that there is an influence between the CMP learning model and conventional learning on mathematical communication skills. Students who get CMP learning get better results than students who get conventional learning.

Students' Mathematical Communication Skills and Levels of Self Esteem

From the calculation results of the ANOVA test for two different groups, the observed F-value ($F_{Obs(B)}$) is 8.52, which is less than the critical F-value ($F_{\alpha(B)}$) of 3.16. Therefore, we can conclude that the null hypothesis H_{0B} is rejected, indicating that there is a significant difference between high, moderate, and low self esteem levels in relation to students' mathematical communication skills. After conducting the Scheffe test for mean comparisons between groups, it was found that students with high self esteem demonstrate better mathematical communication skills compared to students with moderate self esteem. Similarly, students with moderate self esteem exhibit better mathematical communication skills compared to students with low self esteem. Additionally, students with high self esteem show better mathematical communication skills than students with low self esteem.

Students with high self esteem categories are more active than students with moderate self esteem categories. Students with high category self esteem tend to be more able to face learning interactions in class, skilled, and confident in their abilities. Even though they need to be provoked, students with moderate self esteem categories are able to face learning interactions in class and are confident in their abilities. Meanwhile, students with low self esteem are more doubtful and have low self esteem. Students with low self esteem also tend to be passive and avoid social interactions with teachers and other students. Research by Elviani, Sugiatno, and Sayu (2020) states that students with high self esteem relatively have high mathematical communication abilities, while students with moderate self esteem have moderate mathematical communication skills. The results of research by Mahani, Budiyono, and Pratiwi (2019) also state that students with high self esteem have better mathematical communication skills than students with moderate self esteem. This is also reinforced by the research of Rahmayani, Fitraini, and Irma (2022) which states that students in the high self esteem group have very good mathematical communication skills, while students in the moderate self esteem group have mathematical communication abilities in the moderate category.

The Interaction between CMP and Self Esteem

Based on the calculation results of the two-way ANOVA test, the observed F-value ($F_{\mathrm{Obs(AB)}}$) is 4.52, which is greater than the critical F-value ($F_{\alpha\mathrm{(AB)}}$) of 3.16. Therefore, we can conclude that the null hypothesis (H_{0AB}) is rejected, indicating that there is an interaction between the learning model and self esteem on students' mathematical communication skills. The interaction examined in this study is between the CMP learning model and direct learning model, and students' self esteem is categorized as high, moderate, and low.

Various factors influence students' mathematical communication skills, including learning models and self esteem. This study reveals an interaction between the learning model and self esteem in relation to students' mathematical communication skills. The CMP model facilitates the development of students' self esteem at all levels. By engaging with the CMP learning model, students can assess themselves positively, which boosts their self esteem and motivates them to learn mathematics, leading to improved mathematical communication.

In contrast, the direct learning model limits students with high, moderate, and low self esteem, as it revolves around teacher-centered instruction. Students with moderate and low self esteem become less curious and passive, lacking interaction in the classroom. Although students have the opportunity to ask questions and attempt exercises, it is insufficient to foster the development of mathematical communication skills. Consequently, students remain passive and refrain from asking questions. This poses challenges for teachers to gauge students' understanding of the material, hampering the proper development of mathematical communication skills across all self esteem levels.

Conversely, students who received the CMP learning model, irrespective of their self esteem levels (high, moderate, or low), exhibited superior mathematical communication skills compared to those who received the direct learning model. This aligns with the findings of Nufus (2017), which demonstrate the influence of learning interactions and school level on enhancing students' mathematical communication skills. Saragih and Anim (2018) similarly affirm the interaction between the learning model and students' initial mathematical abilities on their mathematical communication skills.

CONCLUSION

Based on the analysis and discussion of the impact of the CMP learning model on mathematical communication skills in relation to students' self esteem, the following conclusions can be drawn: (1) Students using the CMP model exhibit better mathematical communication abilities compared to those using the direct learning model. (2) Students with high self esteem demonstrate superior mathematical communication skills compared to students with moderate self esteem, while students with moderate self esteem have better skills than those with low self esteem. Additionally, students with high self esteem exhibit better mathematical communication skills than students with low self esteem. (3) There is an interaction between the learning models and self esteem in influencing students' mathematical communication abilities. Students with high, moderate, and low self esteem who receive the CMP learning model display better mathematical communication skills than those with similar self esteem levels who receive the direct learning model.

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Mathematics Teachers' Views on Online Learning Implementation Barriers

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Abstract

Mathematics teachers face challenges when it comes to delivering instruction, particularly on topics that require practical applications. This study aimed to determine the barriers encountered in implementing online learning during the pandemic. A quantitative descriptive research design was employed, utilizing a survey technique, with twelve mathematics teachers from St. Paul University Surigao as participants. The primary instrument used to gather data was an adapted questionnaire from Mailizar et al. (2020). Statistical tools employed in this study included Frequency Count and Percentage Distribution, Mean and Standard Deviation, and Analysis of Variance. Findings revealed that respondents poorly perceived the identified online learning implementation barriers, specifically, the teacher-level and student-level barriers. Consequently, the school provided them with enough support for the online learning implementation during the pandemic. It is also concluded that the teachers' number of years in service affects the perception of the online learning implementation barrier in terms of the teacher level barriers. Based on the results, it is recommended that additional training and support be provided to both teachers and students to create an effective online teaching and learning environment, as the teacher-level and student-level barriers were found to be most prominent.

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INTRODUCTION

Louis-Jean and Cenat (2020) stated that the COVID-19 pandemic had altered every aspect of life worldwide. It affected the global economy, health care systems, social life, and most importantly, education. The teachers changed their teaching pedagogy from the physical walls of the classroom [face-to-face learning] to web-based distance learning.

St. Paul University Surigao implemented a full online distance learning modality. SPUS pursued a more adaptive and inclusive Remote Flexible Learning Experience (ReFLEx) where instructional processes, assessment methods, and student advancement programs can be done synchronously or asynchronously and shall be accomplished mainly through online learning platforms. This was included in the Basic Education Learning Continuity Plan of the university as submitted and approved by the Department of Education.

Through the implementation of ReFLEx, SPUS' goal to continually serve the Paulinians in the pursuit of academic excellence and spirituality and to provide more adaptive and inclusive learning became possible. SPUS also used Quipper as their learning management system. Garcia et al. (2022) found that teachers highly utilized Quipper as Learning Management System in teaching in terms of Sending Assignments and Practice Examinations, Creating Educational Content, and Viewing and Downloading Analytics. The Junior High School has been using Quipper for four years, Senior High School for three years, while Grade School for two years.

However, the school faced opportunities and challenges in its implementation. One of the major challenges that teachers, parents, and students face is internet connectivity. Other concerns that the students gave through the survey include difficulty understanding higher concepts, their preference to have classroom classes instead of online, lack of social interaction, and lack of motivation. These concerns corroborate with Alawamleh, Al-Twait, and Al-Saht (2020), who found that students still preferred classroom classes due to the many problems they face. These include lack of motivation, understanding of the material, decreased communication levels, and feelings of isolation caused by online classes. Moreover, students feel that lack of community, technical problems, and difficulties in understanding instructional goals are the major barriers to online learning (Song et al., 2004).

Mailizar et al. (2020) identified barriers that mathematics teachers view as significant to e-learning use during the pandemic. Mathematics teachers are challenged in terms of delivering instruction, especially on topics that require practical applications. With this, the researchers conducted this study to determine the online learning implementation barriers during the pandemic as perceived by the high school mathematics teachers of St. Paul University Surigao. This study also looked into the profile of the respondents in terms of age, sex, number of years in service, highest educational attainment, devices used for online learning, and internet connection. A significant degree of variance between the respondents' profile and their perceptions of the online learning implementation barriers was also tested. The findings will also serve as an additional basis for future projects to be implemented by the university, especially on online learning implementation.

RESEARCH METHODS

This study employed a quantitative research design using a survey technique. This design is deemed appropriate because descriptive research design is a scientific method in which the behavior of a subject is observed and described without influencing him in any way; it is also a valid method for researching specific topics and as a precursor to more quantitative studies (Shuttleworth, 2008 as cited by

Lucero, Guerra, & Arpilleda, 2022). By using this research design, the researchers will be able to determine the online learning implementation barriers during the pandemic as perceived by the high school mathematics teachers.

The respondents of this study were the high school Mathematics teachers from St. Paul University Surigao. Total population sampling was applied where all of the high school mathematics teachers were taken as respondents.

The researchers used an adapted questionnaire from Mailizar et al. (2020) in this study. It consists of two parts. The first part asked about the profile of the respondents in terms of age, sex, number of years in service, highest educational attainment, the device used for online learning, and internet connection. The second part asked about the online learning implementation barriers in terms of teacher-level barriers, school-level barriers, curriculum level barriers, and student level barriers.

In the analyzing the gathered data, the researchers used Frequency Count and Percentage to present the profile of the respondents, Mean and Standard Deviation to determine the online learning implementation barriers as perceived by the high school mathematics teachers, and Analysis of Variance to test the significant degree of variance in the online learning implementation barriers when grouped according to the respondent's profile.

RESULTS AND DISCUSSION

This study aimed to determine the perceptions of the high school mathematics teachers on the online learning implementation barriers during the pandemic. A quantitative descriptive research design using a survey technique was employed. The respondents of this study were the high school mathematics teachers from St. Paul University Surigao. The main instrument used to solicit information is an adapted questionnaire from Mailizar et al. (2020). The statistical tools used in this study were the Frequency Count and Percentage, Mean and Standard Deviation, and Analysis of Variance.

Based on the analysis done on the data gathered, the findings revealed in this study are as follows.

Table 1 shows the profile of the respondents. As to the age of the respondents, 4 (33.33%) belong to the 25-29 years old bracket, 3 (25.00%) from 35-39 years old, 2 (16.67%) from 20-24 years old, and 1 (8.33%) from 30-34, 50-54, and 55-59 years old. As to the sex of the respondents, 6 (50.00%) were males, and 6 (50.00%) were females. As to the number of years in service, 5 (41.67%) served for 0-3 years, 4 (33.33%) served for 4-7 years, 1 (8.33%) served for 8-11, 24-27, and 32-35 years. As to the highest educational attainment, 8 (66.67%) attained a bachelor's degree, and 4 (33.33%) attained a master's degree. As to the devices used for online learning, 9 (75.00%) used both mobile/ handheld devices and computer/laptops, while 3 (25.00%) used only computer/laptops. As to the internet connection used, 5 (41.67%) used Postpaid Wifi/ Landline Connection, 2 (16.67%) used mobile data and prepaid wifi, and mobile data, prepaid wifi, and postpaid wifi/ landline connection, 1 (8.33%) used only mobile data, prepaid wifi, mobile data, and postpaid wifi/ landline connection, respectively.

Table 1. Profile of the Respondents

Profile Variables	*f* (*n*=12)	%
Age		
20-24 years old	2	16.67
25-29 years old	4	33.33
30-34 years old	1	8.33
35-39 years old	3	25.00
50-54 years old	1	8.33
55-59 years old	1	8.33
Sex		
Male	6	50.00
Female	6	50.00
Number of Years in Service		
0-3 years	5	41.67
4-7 years	4	33.33
8-11 years	1	8.33
24-27 years	1	8.33
32-35 years	1	8.33
Highest Educational Attainment		
Bachelor's Degree	8	66.67
Master's Degree	4	33.33
Device Used for Online Learning		
Computer/ Laptop	3	25.00
Mobile/ Handheld Device and Computer/ Laptop	9	75.00
Internet Connection		
Mobile Data	1	8.33
Prepaid Wifi	1	8.33
Postpaid Wifi/ Landline Connection	5	41.67
Mobile Data and Prepaid Wifi	2	16.67
Mobile Data and Postpaid Wifi/ Landline Connection	1	8.33
Mobile Data, Prepaid Wifi, Postpaid Wifi/ Landline Connection'	2	16.67

Based on Table 2, as to the respondents’ perception of the online learning implementation barriers in terms of teacher-level barriers, the indicator I am not confident in using e-learning during the Covid-19 pandemic got the highest mean (*M*=2.17, *SD*=0.83) which can be verbally interpreted as disagree and qualitatively described as poorly perceived. This means that the respondents are confident in delivering instruction during the pandemic with the use of online learning. This is true because e-learning gives remote accessibility and flexibility.

However, the indicator the use of E-learning during this pandemic is not convenient for me got the lowest mean (*M*=1.50, *SD*=0.90) which can be verbally interpreted as strongly disagree and qualitatively described as very poorly perceived. This means that the respondents are at ease with e-learning.

Table 2. Respondents' perception of the online learning implementation barriers in terms of teacher-level barriers

Indicators	*M*	*SD*	VI	QD
Teacher Level Barriers				
1. I do not have sufficient knowledge and skill to use e-learning during the Covid-19 pandemic	1.83	0.83	DA	PP
2. I am not confident in using e-learning during the Covid-19 pandemic	2.17	0.83	DA	PP
3. I do not have experience in using e-learning	1.75	0.75	DA	PP
4. I believe that the use of e-learning in teaching is not useful during this pandemic	1.58	0.90	SDA	VPP
5. The use of E-learning during this pandemic is not convenient for me	1.50	0.52	SDA	VPP
Average	1.77	0.77	DA	PP

Note: 1.00-1.74 (Strongly Disagree/ Very Poorly Perceived); 1.75-2.49 (Disagree/ Poorly Perceived); 2.50-3.24 (Agree/ Highly Perceived); 3.25-4.00 (Strongly Agree/ Very Highly Perceived).

On average, the respondents' perception of the online learning implementation barriers in terms of teacher-level barriers (*M*=1.77, *SD*=0.77) can be verbally interpreted as disagree and qualitatively described as poorly perceived.

Based on Table 3, as to the respondents' perception of the online learning implementation barriers in terms of school-level barriers, the indicator Because of workload, I do not have enough time to prepare e-learning materials got the highest mean (*M*=2.00, *SD*=0.85) which can be verbally interpreted as disagree and qualitatively described as poorly perceived. This means that the teachers are given the time to prepare instructional materials despite the current set-up and their workloads.

Table 3. Respondents' perception of the online learning implementation barriers in terms of school-level barriers

Indicators	*M*	*SD*	VI	QD
School Level Barriers				
1. My school does not have an e-learning system	1.25	0.45	SDA	VPP
2. My school does not have internet connection	1.33	0.49	SDA	VPP
3. School regulations do not support the use of e-learning during the Covid-19 pandemic	1.25	0.45	SDA	VPP
4. Textbooks are not in line with e-learning use	1.50	0.52	SDA	VPP
5. My school does not provide technical support for e-learning use	1.42	0.51	SDA	VPP
6. Because of workload, I do not have enough time to prepare e-learning materials	2.00	0.85	DA	PP
Average	1.46	0.55	SDA	VPP

Note: 1.00-1.74 (Strongly Disagree/ Very Poorly Perceived); 1.75-2.49 (Disagree/ Poorly Perceived); 2.50-3.24 (Agree/ Highly Perceived); 3.25-4.00 (Strongly Agree/ Very Highly Perceived).

However, the indicators My school does not have an e-learning system, and School regulations do not support the use of e-learning during the Covid-19 pandemic got the lowest means (*M*=1.25, *SD*=0.45) which can be verbally interpreted as strongly disagree and qualitatively described as very poorly perceived. This means that the school has an e-learning system. Garcia et al. (2022) as cited by Arpilleda et al. (2023) found that teachers highly utilized Quipper as Learning Management System in teaching in terms of Sending Assignments and Practice Examinations, Creating Educational Content, and Viewing and Downloading Analytics.

On average, the respondents' perception of the online learning implementation barriers in terms of school-level barriers (*M*=1.46, *SD*=0.55) can be verbally interpreted as strongly disagree and qualitatively described as very poorly perceived.

Table 4. Respondents' perception of the online learning implementation barriers in terms of curriculum-level barriers

Indicators	*M*	*SD*	VI	QD
Curriculum Level Barrier				
1. Learning and teaching resources that are available on the e-learning system are not in accordance with the curriculum	1.50	0.52	SDA	VPP
2. Schools require students' assessments that are not in line with e-learning use	1.42	0.51	SDA	VPP
3. The contents of my subject cannot be taught using e-learning	1.42	0.67	SDA	VPP
4. The contents of my subject are difficult to be taught using e-learning	1.75	0.87	DA	PP
5. The contents of my subject are difficult to be understood by students through e-learning	1.92	0.79	DA	PP
Average	1.60	0.67	SDA	VPP

Note: 1.00-1.74 (Strongly Disagree/ Very Poorly Perceived); 1.75-2.49 (Disagree/ Poorly Perceived); 2.50-3.24 (Agree/ Highly Perceived); 3.25-4.00 (Strongly Agree/ Very Highly Perceived).

Based on Table 4, as to the respondents' perception of the online learning implementation barriers in terms of curriculum level barriers, the indicator The contents of my subject are difficult to be understood by students through e-learning got the highest mean (*M*=1.92, *SD*=0.79) which can be verbally interpreted as disagree and qualitatively described as poorly perceived. This means that the respondents understood the contents of their subject and this helped them in delivering the instruction efficiently and effectively. This is true because the department gives a month-long training to prepare the teachers before the start of classes. This training includes strategies on delivering instruction, competence in delivering online classes, and other related activities that equip the teachers with suitable professional qualifications and pedagogical skills and strategies.

However, the indicators Schools require students' assessments that are not in line with e-learning use and the contents of my subject cannot be taught using e-learning got the lowest means (*M*=1.42, *SD*=0.52 and 0.67, respectively) which can be verbally interpreted as strongly disagree and qualitatively described as very poorly perceived. Despite being the lowest indicators, the respondents still described these indicators as very poorly perceived which means that the assessments required by the school are aligned to the use of e-learning and contents can be discussed using online learning. The school uses online learning to continue delivering instruction amid the pandemic.

On average, the respondents' perception of the online learning implementation barriers in terms of curriculum level barriers (*M*=1.60, *SD*=0.67) can be verbally interpreted as strongly disagree and qualitatively described as very poorly perceived.

Table 5. Respondents' perception of the online learning implementation barriers in terms of student-level barriers

Indicators	*M*	*SD*	VI	QD
Student Level Barriers				
1. My students do not have sufficient knowledge and skill in the use of e-learning	1.92	0.67	DA	PP
2. My students do not have devices (i.e. laptop and tablet) for the use of e-learning	1.75	0.62	DA	PP
3. My students are not interested in using e-learning	1.83	0.39	DA	PP
4. My students do not have internet connection	1.83	0.58	DA	PP
5. My students are not able to access the e-learning system	1.50	0.52	SDA	VPP
Average	1.77	0.56	DA	PP

Note: 1.00-1.74 (Strongly Disagree/ Very Poorly Perceived); 1.75-2.49 (Disagree/ Poorly Perceived); 2.50-3.24 (Agree/ Highly Perceived); 3.25-4.00 (Strongly Agree/ Very Highly Perceived).

Based on Table 5, as to the respondents' perception of the online learning implementation barriers in terms of student level barriers, the indicator My students do not have sufficient knowledge and skill in the use of e-learning got the highest mean (*M*=1.92, *SD*=0.67) which can be verbally interpreted as disagree and qualitatively described as poorly perceived. This means that the students are knowledgeable and skillful enough in the use of e-learning. Before the classes begin, a week-long student orientation is given to students especially on the use of the e-learning platforms. They were taught on how to navigate, especially the learning management system.

However, the indicator My students are not able to access the e-learning system got the lowest mean (*M*=1.50, *SD*=0.52) which can be verbally interpreted as strongly disagree and qualitatively described as very poorly perceived. This means that the learning management used by the school is accessible and student friendly. Quipper gives an annual orientation to the students on Quipper use.

On average, the respondents' perception of the online learning implementation barriers in terms of student level barriers (*M*=1.77, *SD*=0.56) can be verbally interpreted as disagree and qualitatively described as poorly perceived.

Based on Table 6, there is no significant degree of variance between the respondents' age and the online learning implementation barriers in terms of teacher-level, school-level, curriculum-level, and student-level barriers (*p*-values=0.394, 0.411, 0.556, and 0.207, respectively). This means that perceptions of the respondents are not affected by their age. The result contradicts the findings of Esteve et al. (2016) as cited by Arpilleda et al. (2023) that the youngest group had a higher perception of digital competence (between 20 and 24 years old).

Although the concepts being studied are different, both studies look into differences between perceptions and age.

Table 6. Significant degree of variance between the respondents' profile and the online learning implementation barriers

Profile	Dependent	*F*	*p*-values
Age	Teacher Level Barriers	1.24	0.394
	School Level Barriers	1.19	0.411
	Curriculum Level Barriers	0.86	0.556
	Student Level Barriers	2.03	0.207
Sex	Teacher Level Barriers	4.60	0.057
	School Level Barriers	0.11	0.749
	Curriculum Level Barriers	0.04	0.849
	Student Level Barriers	1.69	0.223
Number of Years in Service	Teacher Level Barriers	4.31	0.045*
	School Level Barriers	0.44	0.779
	Curriculum Level Barriers	0.74	0.596
	Student Level Barriers	0.26	0.894
Highest Educational Attainment	Teacher Level Barriers	0.02	0.878
	School Level Barriers	0.37	0.554
	Curriculum Level Barriers	0.40	0.542
	Student Level Barriers	0.03	0.868
Device Used for Online Learning	Teacher Level Barriers	0.02	0.900
	School Level Barriers	0.00	0.971
	Curriculum Level Barriers	0.00	1.000
	Student Level Barriers	0.18	0.683
Internet Connection	Teacher Level Barriers	3.49	0.080
	School Level Barriers	0.16	0.968
	Curriculum Level Barriers	0.57	0.724
	Student Level Barriers	0.23	0.938

Note: * - significant at $p<0.05$

There is no significant degree of variance between the respondents' sex and the online learning implementation barriers in terms of teacher-level, school-level, curriculum-level, and student-level barriers (*p*-values=0.057, 0.749, 0.849, and 0.223, respectively). This means that perceptions of the respondents are not affected by their sex. This concurs with the findings of Mula et al. (2022) that sex of the respondents does not affect their perception. The dominance of male teachers over female teachers in e-learning use is no longer valid (Mailizar, 2020).

There is no significant degree of variance between the respondents' number of years in service and the online learning implementation barriers in terms of school-level barrier, curriculum level barrier, and student level barrier (*p*-values=0.779, 0.596, and 0.894, respectively). However, there is a significant degree of variance between the respondents' number of years in service and the online learning implementation barriers in terms of teacher-level barriers (*p*-value=0.045). This means that perceptions of the respondents, specifically on teacher-level barriers, are affected by their length of service. In the study conducted by Arpilleda et al. (2023)

on Education 4.0, they found out that the length of service of the participants affects how they understand, value, and accept the nature and demands of Education 4.0.

There is no significant degree of variance between the respondents' highest educational attainment and the online learning implementation barriers in terms of teacher-level barrier, school level barrier, curriculum level barrier, and student level barrier (*p*-values=0.878, 0.554, 0.542, and 0.868, respectively). This means that perceptions of the respondents are not affected by their educational attainment. It is generally accepted that more experienced teaching is necessary to build the skills needed for effective teaching (Mailizar et al., 2020).

There is no significant degree of variance between the respondents' devices used for online learning and the online learning implementation barriers in terms of teacher-level barrier, school level barrier, curriculum level barrier, and student level barrier (*p*-values=0.900, 0.971, 1.000, and 0.683, respectively). This means that perceptions of the respondents are not affected by their devices used for online learning.

There is no significant degree of variance between the respondents' internet connection and the online learning implementation barriers in terms of teacher-level barrier, school level barrier, curriculum level barrier, and student level barrier (*p*-values=0.080, 0.968, 0.724, and 0.938, respectively). This means that perceptions of the respondents are not affected by their internet connection.

CONCLUSION

Based on the findings gathered, it is concluded that the St. Paul University Surigao provided the teachers with enough support for the online learning implementation during the pandemic as the teachers very poorly perceived the identified online learning implementation barriers. It is also concluded that the teachers' number of years in service affects the perception of the online learning implementation barrier in terms of the teacher level barriers.

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Critical Thinking Ability as a Correlate of Students' Mathematics Achievement: A Focus on Ability Level

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Abstract

This study investigated the correlation between critical thinking ability and academic achievement in mathematics among 200 Senior Secondary School Two (SS2) students in Orlu Education Zone in Imo State, Nigeria. The researchers utilized a correlational design and employed a multi-stage but simple random sampling technique to select the sample. Data were collected using two instruments: the Watson-Glaser Critical Thinking Appraisal (WGCTA) and the Mathematics Achievement Proforma (MAP). The validity of the WGCTA instrument was ensured through expert suggestions and guidance. The study addressed three research questions and tested three null hypotheses using the Pearson Product Moment correlation coefficient at a significance level of 0.05. The findings of the study indicated a low positive and significant correlation between critical thinking ability and academic achievement in mathematics for all students. However, there was no significant relationship between critical thinking ability and achievement among high achievers, while there was a significant but negligible positive relationship between critical thinking ability and achievement among low achievers. The study recommended that mathematics teachers should incorporate critical thinking development in their classroom instruction to enhance the critical thinking ability of students, which can lead to improved academic achievement, especially among low achievers.

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INTRODUCTION

In the current era, there is an urgent need to emphasize on actualization of scientific and technological developments because of their global advancement. As a result, students are being encouraged to develop an interest in and pursue science-related subjects and courses since they will exponentially increase the level of growth in these critical fields in the near future. At all educational levels, mathematics is a fundamental prerequisite for learning any science discipline. Mathematics has significantly helped the development of science and technology for thousands of years and will continue to do so now (Algani, 2022). Students must have at least a credit in mathematics on their Senior Secondary School Certificate Examination in order to be admitted to a tertiary institution. These days, mathematics is used in practically every sphere of human endeavor, and it is essential to a nation's economic growth.

As Nigeria matches to prioritize economic growth, we need nothing short of good academic achievement in mathematics at all educational levels. The good academic performance of students at the senior secondary school is of paramount importance in every educational system (Brew, Nketiah, & Koranteng, 2021). Despite the importance of mathematics, observations and reports from West African Examination Council Chief Examiners revealed that Nigeria Secondary Students continue to perform poorly in mathematics examinations. According to the 2021 West African Examinations Council (WAEC) May/June results, only 32.03% of Nigerian secondary school students obtained credit passes in Mathematics (WAEC, 2021). The low performance in mathematics has been a persistent problem in Nigeria, as highlighted by the National Examination Council (NECO) which reported that only 26.01% of candidates obtained credit passes in Mathematics in 2020 (NECO, 2020). The students' poor performance indicates that they are either not being taught mathematics correctly in schools or are having problems understanding the ideas and skills or may have learning difficulties in acquiring mathematics though-processes, concepts and inability to think critically

In terms of critical thinking, Adebiyi and Akintoye (2020) found a positive correlation between students' critical thinking skills and their performance in mathematics. The study, which was conducted among senior secondary school students in Ogun State, Nigeria, recommended that mathematics teachers should incorporate critical thinking into their instructional strategies to enhance students' performance in the subject. Similarly, another study by Olanrewaju and Ogundipe (2021) examined the effects of a critical thinking instructional model on senior secondary school students' achievement in mathematics and found that the model significantly improved students' performance. The study recommended the adoption of the model in mathematics instruction in Nigerian schools.

The problem with ability level in critical thinking and mathematics achievement is that not all students possess the same level of critical thinking ability, and this can impact their academic performance in mathematics. Some students may struggle to apply critical thinking skills when solving mathematical problems, while others may excel in this area. This can lead to disparities in academic achievement based on students' ability level in critical thinking, particularly in a subject like mathematics that requires strong problem-solving skills. This issue highlights the need for mathematics teachers to incorporate critical thinking development into their instructional strategies to support students' learning and enhance their academic achievement, regardless of their ability level (Ogunniyi, Jegede, & Ogundipe, 2019). By doing so, teachers can help students develop the necessary skills to excel in mathematics and other subjects, and prepare them for success in higher education and beyond

The ability to think critically, analyze, solve problems and make decisions has been the foundation for the success and progress of the human race. For students to perform and contribute successfully in the society they have to possess competencies on these abilities. Educational institutions have a major responsibility to provide the tools and learning opportunities that enable students to develop these abilities. One major school subject through which critical thinking ability could be inculcated to students is mathematics, which is in conformity with the National Educational Policy goals to imbibe the ability to make rational decisions. Similarly, the STEM framework does have objectives that correspond with the characteristics of 21st century-education, among which are critical thinking skills, or thinking that is always curious about the information available in order to achieve a thorough understanding (Setyawati et al., 2022). Since mathematics is a subject that is also based on logic and one has to make sense of concrete as well as abstract problems, then it can be argued that it can enhance critical thinking ability where logic is one of its components.

The concept of critical thinking and its development over centuries of literature is well documented. John Dewey defined critical thinking as "reflective thinking," emphasizing active and persistent consideration of beliefs and knowledge in light of evidence (Dewey, 1933). Fisher further elaborated on Dewey's definition, highlighting the importance of reasoning and implications of reasoning in shaping one's beliefs (Fisher, 2001). Edward Glaser expanded on Dewey's definition, emphasizing the need for evidence and a disposition to use critical thinking skills (Glaser, 1984). Ennis took Dewey's definition further by including decision-making as a component of critical thinking (Ennis & Norris, 1990), while Paul considered metacognition as a crucial element (Paul, 1995). Overall, critical thinking involves thoughtful consideration of problems and evidence to arrive at logical conclusions, with a disposition to use critical thinking skills and remain cognizant of one's own thinking process. (Dewey, 1933; Fisher, 2001; Glaser, 1984; Ennis & Norris, 1990; Paul, 1995).

The most crucial aspects of critical thinking and intellectual standards were presented by Michael Scriven and Richard Paul in the Annual International Conference on Critical Thinking and Education Reform. They describe critical thinking as the intellectual process of actively conceptualizing, applying, analyzing, synthesizing, and/or evaluating information gathered from, or generated by, observation, experience, reflection, reasoning, or communication, as a guide to belief and action. In its exemplary form, it is based on universal intellectual values that transcend subject matter divisions: clarity, accuracy, precision, consistency, relevance, sound evidence, good reasons, depth, breadth, and fairness (Scriven & Paul, 1987).

According to the above literatures, critical thinking involves logical reasoning and ability to separate facts from opinion, examine information critically with evidence before accepting or rejecting ideas and questions in relation to the issue at hand. In other words, it encourages people to reflect, raise doubts about things, challenge preconceived notions, come up with answers to situations, and make wise choices when faced with difficulties. For this study critical thinking requires a certain mindset which is essentially evaluative in nature. It is the ability to think clearly and rationally.

Critical thinking is connected to mathematics, scientific problem solving and physical science among other fields. In mathematics, critical thinking is referred to as the ability to evaluate the presented mathematical problems and think about vital skills that allows one to tackle the problems efficiently. The vital skills include; clarity of thought, intellectual integrity, problem identification and solution, respect for evidence, internal coherence, intellectual standards, meta-cognition, questioning, deductive and inductive reasoning, argument mapping and ethical reasoning, to name a few (Ennis, 1993; Fisher, 2001; Paul & Elder, 2004; Scriven & Paul, 1987). Mathematics is also the logical language for expressing ideas of shapes, quantities, size, order, change and dynamism in systems and for explaining the complexities of modern society in the business, economic, academic, engineering, medical settings and other fields. The teachers are responsible to equip students with these skills and generate interest in learning mathematics as a means of acquiring effective tools for problem solving in life.

Mathematics education is a field of study that focuses on teaching methods, curriculum development, and instructional strategies to promote mathematical learning. It also involves the study of how students learn mathematics, the development of assessment tools, and research on enhancing problem-solving and critical thinking skills in mathematics (National Council of Teachers of Mathematics, 2021). The story of mathematics education in Nigeria is the story of our inherited western education system. To teach mathematics in an understandable way to our students has always been a problem and so different methods are investigated and practiced for better achievement by students in mathematics in the past years. Critical thinking is considered important in the academic fields especially in mathematics because it enables one to analyze, evaluate, explain and restructure their thinking, thereby decreasing the risk of adopting, acting on or thinking with, a false belief (UKEssays, 2021). If mathematics teachers incorporate critical thinking teaching techniques, teaching and learning may be improved which will transcend to students' academic achievement in mathematics.

Academic achievement represents performance outcomes that indicate the extent to which a person has accomplished specific goals that were the focus of activities in instructional environments, specifically in school, college, and university (Kane, 2017). In the school setting, it is referred to as evaluation of students' learning outcomes as quantified on the basis of marks or grades in school subjects. Kane further stresses that School systems mostly define cognitive goals that either apply across multiple subject areas (e.g., critical thinking) or include the acquisition of knowledge and understanding in a specific intellectual domain (e.g., numeracy, literacy, science, history). Academic achievement is commonly measured by examinations or continuous assessment. In this study, academic achievement was defined according to how well a student accomplished work in the school setting in mathematics. It was assessed by mathematics teachers and represented by students' cumulative grade for the terms. The marks or scores assigned by teachers could be high or low (ability level), which means that academic achievement, could either be good or bad.

There is a common saying that an examination is not a true test of knowledge. This implies that where the evaluation of students lacks validity and prejudice it does not guarantee that high achievers could exhibit high critical thinking ability. Students are divided into groups according to their performance on a test or examination, which determines their ability level. Ability level relates to a student's demonstration of skill, attitudes, and knowledge in relation to learner outcomes for the grade level. Regarding a teaching-learning setting, there are three different types of ability levels: low achievers, medium or average achievers, and high achievers (Talca, 2007). In categorization of students' academic ability levels, Talca (2007) grouped students with (0-49%) scores as low ability, (50-64%) scores as medium ability and (65-100%) as high ability level.

Low achievers are individuals whose academic potentials are estimated to be below average while their performance is characterized as poor, and they need constant study, assessment, and assistance in order to improve their situation. Medium or average achievers are individuals whose academic performance are estimated to be average, not a reflection of their competency, but rather, in part, a reflection of their inability to put up the additional work required to get better results. As a result, they limit their learning to the classroom and refrain from seeking out other sources of information. Higher achievers are individuals whose performance is deemed to be good and whose academic potential is above the class average. In the context of this study however, ability level was classified as high achievers (65-100%) and low achievers (0-49%) only, in mathematics cumulative grade for the terms. The ability level classification into high achievers (65-100%) and low achievers (0-49%) is based on the grading system used by the National Examinations Council (NECO) and the West African Examinations Council (WAEC) in Nigeria. Scores of 65% and above are considered distinctions, while scores below 50% are failing grades. This classification is widely recognized and used in Nigeria

Students categorized as higher achiever may or may not exhibit higher critical thinking skills than those students categorized as lower achiever. Looking into secondary school classroom settings, mathematics in secondary schools is mainly characterized by a high percentage of low achievers as evidenced by the poor achievement of the students in mathematics examinations in public secondary schools. According to a study conducted by Adeyemi and Adeyemo (2019) in Nigeria, mathematics education in secondary schools is characterized by a high percentage of low achievers. The study found that many students performed poorly in mathematics examinations in public secondary schools, with some even failing the subject. Findings indicate moderate positive correlation between academic achievement and critical thinking ability in undergraduate students (Braun, & Clark, 2021). Findings have proved that critical thinking is significantly and positively correlated with academic achievement (Chiketa & Okigbo, 2021; Fitriani et al., 2020; Nur'Azizah, Utami, & Hastuti, 2021; Nwuba, Egwu, & Osuafor, 2022). Studies that assess students' critical thinking ability reveal that students often fail in tasks that require critical thinking (Fadhlullah & Ahmad, 2017; Flores et al., 2012; Yasir & Alnoori, 2020).

The body of literature related to critical thinking continues to grow, there continues to remain disagreement of factors that are associated with the ability to think critically. The current research may differ from previous studies in terms of the specific research questions, sample population, research design, and data analysis methods. This study aims to provide a better understanding of this relationship and identify specific critical thinking skills associated with academic achievement. The results of this research can inform the design of effective interventions and strategies, guide targeted interventions, and inform educational policies to prepare students for success in the modern world. The research can achieve new things such as providing a better understanding of critical thinking and academic achievement in mathematics, identifying critical thinking skills that need improvement, and increasing awareness of the importance of critical thinking skills in mathematics education. All of these can help develop better educational interventions and policies and prepare students for success in the modern world.

It is against this background that this study sought to establish the extent of correlation between critical thinking ability and students' academic achievement in mathematics focusing on ability level.

Research Questions

The study was guided by three research questions. Firstly, what is the correlation between critical thinking ability and students' achievement in mathematics? Secondly, what is the relationship between critical thinking ability and achievement among high achievers in mathematics? Finally, what is the relationship between critical thinking ability and achievement among low achievers in mathematics?

Null Hypotheses

The following null hypotheses were tested at 0.05 level of significance: Firstly, there is no significant correlation between critical thinking ability and students' achievement in mathematics. Secondly, there is no significant relationship between critical thinking ability and the achievement of high achievers in mathematics. Finally, there is no significant relationship between critical thinking ability and the achievement of low achievers in mathematics.

RESEARCH METHODS

The study used correlational survey research design, which aimed at correlating critical thinking and students' academic achievement in mathematics. It seeks to establish the relationship that exists between two or more variables. The population of the study comprised all four thousand eight hundred and fifty-four (4854) senior secondary school class two (SS2) in forty (40) government owned secondary schools in Orlu Education zone one (1) in Imo State, Nigeria. The participants consisted of two hundred (200) SS2 students. The researchers adopted the multi-stage but simple random sampling technique to draw the sample. Two instruments were used to collect data for this study. They include: Watson-Glaser Critical Thinking Appraisal (WGCTA) and Mathematics Achievement Proforma (MAP). The WGCTA is a widely used standardized instrument of critical thinking ability that measures the ability to think critically across five cognitive domains: inference, recognition of assumptions, deduction, interpretation, and evaluation of arguments. The WGCTA consists of 40 test questions that assess critical thinking ability across the five cognitive domains. MAP was used to collect data on students' achievement scores in mathematics. It includes the cumulative scores of students' first and second term mathematics results. Two experts validated and modified the instrument (WGCTA) to ensure the appropriateness of the language used and coverage content is suitable in eliciting the required information. A single-administration reliability and Kuder-Richardson formula 21 (KR-21) reliability coefficient of 0.83 was obtained, WGCTA was deemed reliable. Mathematics Achievement Proforma (MAP) of high achievers (65-100%) and low achievers (0-49%) in mathematics cumulative grade for first and second terms were used. The data obtained were analyzed using Pearson Product Moment correlation coefficient in answering research questions and *t*-test in testing null hypothesis at 0.05 level of significance with aid of statistical package for Social Sciences (SPSS) version 20. Decision rule for correlation coefficient was adopted from Best and Kahn (2013) who provided the following rules for judging the strength of correlation between two variables.

Table 1. Relationship Category

Correlation (*r*)	Relationship
.00 to .20	Negligible
.20 to .40	Low
.40 to .60	Moderate
.60 to .80	Substantial
.80 to 1.00	High to very high

The decision to reject or accept a null hypothesis was based on the probability value (*p*-value) and the 0.05 significance level. Where the *p*-value is less than 0.05 alpha level, the null hypothesis is rejected, if otherwise not rejected.

RESEARCH RESULTS

The results are presented in the order of the research questions and hypotheses raised.

Table 2. Correlation between Critical Thinking Ability and Students' Achievement in Mathematics

Variable	*N*	*R*	*R*-square	*Sig.*	Decision
WGCTATEST	200	$.260^{**}$	0.068	.000	Significant
Achievement	200				
**correlation is significant					

The results in Table 2 from the correlation analysis, the correlation coefficient (*r*) of 0.260 was obtained and the null hypothesis ($H0_1$) is rejected because *p*-value (*Sig.*=0.000) is less than the 0.05 level of significance. This shows a low positive and significant correlation between critical thinking ability and students' academic achievement in mathematics. By squaring the correlation coefficient and multiplying by hundred (100), we obtained 6.8%. This also shows that only 6.8% of variation in critical thinking ability. That is, critical thinking ability influences achievement in mathematics to the tune of 6.8%.

Table 3. Relationship between Critical Thinking Ability and Achievement of High Achievers in Mathematics

Variable	*N*	*R*	*R*-square	*Sig.*	Decision
WGCTATEST	41	.169	0.0286	.292	Not Significant
Achievement	41				
**correlation is significant					

The results in Table 3 revealed that the correlation coefficient (*r*) is 0.169, which is positive but negligible and null hypothesis ($H0_2$) is accepted because *p*-value (*Sig.*=0.292) is greater than the 0.05 level of significance. Therefore, there is negligible positive and no significant relationship between the critical thinking ability and achievement of high achievers in mathematics. The result also shows that critical thinking ability of high achievers influences their mathematics achievement to turn of 2.86%.

Table 4. Relationship between Critical Thinking Ability and Achievement of Low Achievers in Mathematics

Variable	*N*	*R*	*R*-square	*Sig*.	Decision
WGCTATEST	159	.161**	0.0259	.043**	Significant
Achievement	159				
**correlation is significant					

In table 4, the results of the correlation analysis revealed that the correlation coefficient (r) is 0.161, which is negligible positive and null hypothesis (HO_3) is accepted because p-value (*Sig*.=0.043) is less than the 0.05 level of significance. Therefore, there is negligible positive and significant relationship between the critical thinking ability and achievement of low achievers in mathematics. Further computation shows that 2.59% of the variation in mathematics achievement of low achievers could be explained by variation in their critical thinking.

DISCUSSION

The result indicates that the knowledge of critical thinking ability of students could not accurately correlate their achievement in mathematics. The result also shows that critical thinking ability and mathematics achievement change in the same direction. This is an indication that increase in critical thinking ability leads to increase in mathematics achievement and vice versa. This is in agreement with the findings of the previous studies (Chiketa & Okigbo, 2021; Fitriani et al., 2020; Nur'Azizah et al., 2021; Nwuba et al., 2022). The previous studies also found a significant positive relationship between critical thinking ability and academic achievement, this study found a low correlation between the two variables while Braun, and Clark (2021) found a moderate relationship. The findings further revealed that critical thinking ability uniquely contributed only 6.8% of the variance in mathematics achievement. This is not comparable with the report of 18% by Scott and Bryan (2006). This is an indication that there is an inconclusive result on the extent of correlation between critical thinking ability and academic achievement. Therefore, the critical thinking ability does not absolutely influence achievement in mathematics as suggested by the findings.

The findings also showed there is negligible positive and no significant relationship between the critical thinking ability and achievement of high achievers in mathematics. while there is negligible positive and significant relationship between the critical thinking ability and achievement of low achievers in mathematics. This also supports earlier submission that knowledge of students' critical thinking ability could not accurately predict their achievement in mathematics, irrespective of whether the students are classified as high or low achievers. Further computation also revealed that 2.86% and 2.59% of the variation in mathematics achievement of high and low achievers respectively, could be explained by variation in their critical thinking ability. The remaining 97.1% and 97.4% respectively, is accounted for by chance. Therefore, one can rightly say that students' critical thinking ability does not account for their achievement in mathematics. The implication for these results is that students have not yet developed the critical thinking ability to handle mathematical problems. This is true for both high and low achievers. Scott and Bryan (2006) found that students categorized as high achievers exhibit higher critical thinking skills than those students categorized as low achievers. These findings are not in accordance with the findings of the study as related to their achievement in mathematics. Thus, critical thinking ability plays little or no role in the mathematics achievement of the high achievers.

CONCLUSION

Based on the discussion of the findings, the following conclusions were drawn that there is a low positive and significant correlation between critical thinking ability and achievement of students in mathematics; the knowledge of critical thinking ability of the students could not accurately predict their achievement in mathematics; the critical thinking ability plays little or no role in the mathematics achievement of high achievers.

The following recommendations were made: (1) Mathematics teachers should strive to incorporate critical thinking development in their classroom presentation, (2) Education practitioners should facilitate the development of critical thinking ability through seminars, and workshops.

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Some Elementary Combinatory Properties and Fibonacci Numbers

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Abstract

In general, in the midst of History of Mathematics textbooks, we are faced with a discussion due to curiosity about the emblematic Fibonacci Sequence, whose popularization occurred with the proposition of the reproduction model of immortal rabbits. On the other hand, in the comparison of the multiple approaches and discussions of certain subjects in Elementary Mathematics, in the present work, we highlight combinatorial interpretations that, with the support of a characteristic and fundamental reasoning for the mathematics teacher, can be generalized and formalize some eminently intuitive components. In particular, this work deals with properties derived from the notion of tiling and decomposition of an integer that, depending on the board, will correspond to the numbers of the Fibonacci Sequence. We bring a theoretical discussion supported by great names that research in this area.

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INTRODUCTION

Significant traces of Combinatorics principles can be found in several civilizations and remote times. In this primitive scenario, the authors Wilson and Watkins (2013) indicate multiple meanings, such as: energetic, poetic, mystical, educational, etc. We have an example in Figure 1, where the authors discuss manuscripts by Ramon Llull, Duke of Venice, in 1210.

In this example, a chapter of the work *Ars Compendiosa Inveniendi Veritatem* (The Concise Art of Finding the Truth, English translation) began by listing sixteen attributes of God: goodness, greatness, eternity, power, wisdom, love, virtue, truth, glory, perfection, justice, generosity, mercy, humility, sovereignty, and patience. So, Ramon Llull wrote, combinatorially $C_{16,2} = 120$ short essays of about 80 words each, considering God's goodness related to greatness (Wilson & Watkins, 2013).

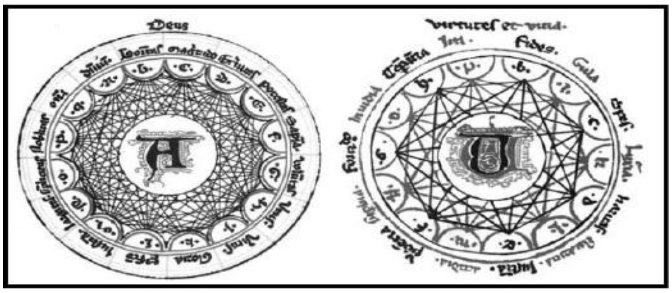

Figure 1. Wilson and Watkins (2013) recover primitive concepts that gave rise to Combinatorics.

As a contemporary motivation on the use of Combinatorics, and for a preliminary discussion, let us consider particular decompositions of the positive integer '7' Indeed, trivially, we have: $\{1+1+1+1+1+1+1;1+2+2+2;1+3+3;$ $1+2+4;1+6;\ 3+1+3;\ 7;\ ...\}$. In the previous set we can see some examples called *compositions of the number 7*. On the other hand, when we examine the order of the terms, we can write $7=1+6$ or $7=6+1$, which represent different compositions for the positive integer $n=7$. Furthermore, compositions of the particular type or species may occur $7=3+1+3$, which represents the same composition, in any of the senses that we consider. When this occurs, we say we have a palindrome (Grimaldi, 2012). Let's see another example in more detail.

In fact, we can say that there are 16 ways to write the positive integer $n=5$ as the sum of integers (see Figure 2). Note that when order is not relevant, compositions $5=4+1=1+4$ are considered equal. In any case, following the reasoning employed by Grimaldi (2012), we seek to determine a formula for the number of compositions of a positive integer $n \in \mathrm{N}$. In a heuristic way, the author advises to consider a trivial composition for the integer $n=5=1+1+1+1+1$, counting the number of installments (5) and the number of additive operations (4 times) (Figure 2):

(1) 5	(5) $2+3$	(9) $2+2+1$	(13) $1+2+1+1$
(2) $4+1$	(6) $3+1+1$	(10) $2+1+2$	(14) $1+1+2+1$
(3) $1+4$	(7) $1+3+1$	(11) $1+2+2$	(15) $1+1+1+2$
(4) $3+2$	(8) $1+1+3$	(12) $2+1+1+1$	(16) $1+1+1+1+1$

Figure 2. Grilmaldi (2012) discuss the compositions of the positive integer $n=5$.

Intuitively, and when comparing with the data in Figure 2, for the set $\{1,2,3,4\}$ we could determine that $2^4=16$ represents the number of subsets of all partitions. On the other hand, if we take a subset $\{1,3\}\subset\{1,2,3,4\}$, we can form the following compositions: $\left(1\underset{1^a}{+}1\right)+\left(1\underset{3^a}{+}1\right)+1$ and $\left(1\underset{1^a}{+}1\right)+\left(1\underset{3^a}{+}1+1\right)$ considering the position of additive operations '+' in 1st and 3rd position in the decomposition.

Grimaldi (2012) observes that the subset indicates that parentheses should be placed around the '1' in the 1st and 3rd positions, where addition operations occur.

Still on Figure 2, we identify the subset of compositions that involve only the digits '1' and '2' in the composition of the positive integer $n=5$. Note that if we eliminate all compositions (8 compositions), except those with the digits '1' and '2', we can determine $2^4-8=16-8=8=F_6$ (later, Table 1). We can see (Figure 2) that only two palindromes occur:

$$2+\underbrace{1}_{\text{central term}}+2,\quad 1+1+\underbrace{1}_{\text{central term}}+1+1.$$

We can easily observe the existence of central terms in both palindromes. In Figure 3 we can see the example of a palindrome, with an odd central term, within the case of integer compositions $n=5$.

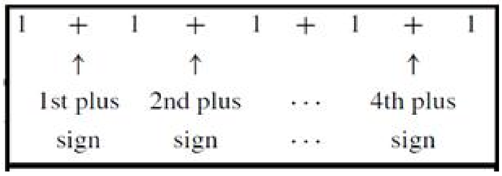

Figure 3. Grilmaldi (2012) discuss the compositions of the positive integer $n=5$ considering the operations.

Before proceeding to the subsequent sections, it is essential to point out the considerations of De-Temple and Webb (2014), when they examine some standard procedures in solving combinatorial problems, namely: (i) introduce a notation, with respect to which h_n represents an answer to be determined in the n-th case; (ii) determine some particular initial values h_1, h_2, h_3, h_4, etc. by direct count; (iii) employ combinatorial reasoning in order to determine the recurrence relation that expresses h_n depending on the previous values of the sequence; (iv) solve the recurrence relation in order to find a unique solution.

In this work, we seek to carry out a bibliographical survey that supports a mathematical discussion about some combinatorial problems that have varied interpretations, based on works of great names within the research in this field of knowledge such as Benjamin and Quinn (1999; 2003), Hemenway (2005), Koshy (2001; 2019), Grimaldi (2012), Singh (1985) and Vajda (1989). His works relate to the emblematic Fibonacci Sequence and its development. Such an approach is usually neglected by History of Mathematics textbooks, which place too much emphasis only on its anecdotal aspects and a bias that does not involve a content that overcomes curiosity for the history of the production of the 'immortal rabbits' (Figure 4).

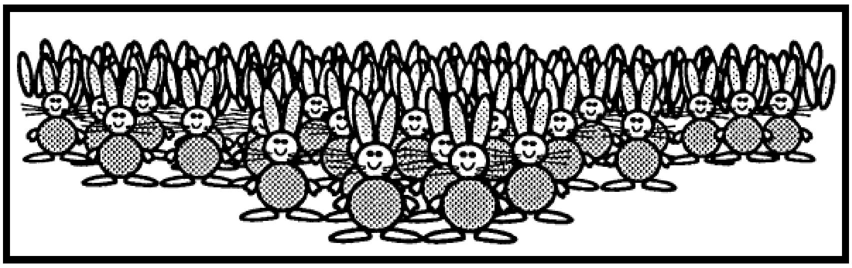

Figure 4. Gullberg (1997) discusses approaches to the Fibonacci Sequence.

Given the above, we seek to discuss the following question: “What elementary properties of Combinatorics allow a meaning and/or interpretation for the set of numbers that occur in the Fibonacci Sequence?” Thus, in the following sections we bring a theoretical discussion based on a mathematical approach to the subject.

COMPOSITIONS, PALINDROMES AND FIBONACCI NUMBERS

In the introductory section we found, in a heuristic way, that there is a correspondence involving the set of compositions of the positive integer $n=5$ and the subsets of the set $\{1,2,3,4\}$, which correspond to the arithmetic expression $2^4=16$ and that represents the subset quantity of all your partitions. Inductively, according to Grimaldi (2012, p. 25), we could write that, “given a positive integer $n\in\mathrm{N}$, we can determine the number 2^{n-1} as the total of compositions”.

However, how do the previous properties relate, more precisely, to the Fibonacci sequence defined by the relation $F_{n+1}=F_n+F_{n-1}$, with initial values $F_0=0$ and $F_1=1$?

About the Fibonacci Sequence and Some Elementary Properties

For the purpose of the problem that we seek to discuss, let us consider the recurrence $F_{n+1}=F_n+F_{n-1}$ and the initial values $F_0=0$ and $F_1=1$, that determine the values in Table 1.

Table 1. Description from the recurrence of the Fibonacci numbers, for $F_n, n\geq 1$

F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	**...**	F_n
0	1	1	2	3	5	8	13	21	34	55	89	144	233	...	...

Strictly speaking, let's consider compositions of a positive integer $n\in\mathrm{N}$, considering only when only the digits '1' and '2' occur. To exemplify, let's consider Figure 5, suggested by Grimaldi (2012). Indeed, we determine some compositions of the positive integers $n=3,4,5,$ as we can see:

$$\begin{array}{ll}(n=3): & 2+1,\ 1+2,\ 1+1+1\\ (n=4): & 2+2,\ 2+1+1,\ 1+2+1,\ 1+1+2,\ 1+1+1+1\\ (n=5): & 2+2+1,\ 2+1+1+1,\ 1+2+1+1,\ 1+1+2+1,\\ & 1+1+1+1+1,\\ & 2+1+2,\ 1+2+2,\ 1+1+1+2\end{array}$$

Figure 5. Grimaldi (2012) discuss compositions of the integers $n=3,4,5$ with only digits ‘1’ and ‘2’.

Grimaldi (2012) defines the term c_n which it interprets as the number of compositions, when the digits '1' and '2' occur. From an arithmetic point of view and with the support of Figure 5, we can determine, as an arithmetic sum, that $c_5=8=F_6=5+3=c_4+c_3$. Grimaldi (2012, p. 25) explains that “the first five compositions of $n=5$ can be obtained by adding the term '+1', from the five previous compositions of $n=4$.

The previous argument supports the definition of recurrence $c_n = c_{n-1} + c_{n-2}$, $n \geq 3$, where we can easily understand that $c_1 = 1$ and $c_2 = 2$ correspond to the number of compositions of the integers $n = 1, 2$, $c_1 = 1$ and $c_2 = 2$, respectively. Furthermore, without further details, Grimaldi (2012) assumes that $c_n = F_n$, $n \geq 1$. It should be noted that in the previous examples we can determine the number of palindromes present in each decomposition. Indeed, if we seek to determine the compositions of the integer $n = 11 \therefore c_{11} = F_{12} = 144$, for example. In this case, for the odd integer $n = 11$, when we consider their decompositions in terms of '1' and '2', each palindrome must contain an odd central term. For example, when we write:

$$1+1+1+1+1+\underbrace{1}_{\text{central term}}+1+1+1+1+1,$$

because the digit '1' is the only possibility for a central term in the palindrome. Grimaldi (2012) observes that, for this palindrome we can consider, the set of compositions of the integer $n=5$, that correspond to the value $n = 5 \therefore c_5 = F_{5+1} = F_6 = F_{\frac{11+1}{2}} = 8$. Furthermore, for this set of 8 compositions, with the central term fixed at '1', we determined all the palindromes present in the decomposition of the integer, when considering the digits '1' and '2'.

From this particular case, Grimaldi (2012, p. 26) states that "in general, if the positive integer n is odd, when we consider the set of compositions $c_n = F_{n+1}$, we determine that $F_{\frac{n+1}{2}}$ correspond precisely to the set of palindromes".

On the other hand, when dealing with an even integer we have, for example, $n = 12 \therefore c_{12} = F_{13} = 144$. Let's take the palindrome:

$$[2+2+2]\underbrace{+}_{\text{central term}}[2+2+2]$$

On the right side, we see a composite of the integer $n = 6 = 2+2+2$ for which the following correspondence holds $n = 6 \therefore c_6 = F_{6+1} = F_7 = 13$. However, compositions (palindromes) of the form:

$$1+1+1+1+1+\underbrace{2}_{\text{central term}}+1+1+1+1+1$$

for the positive integer $n = 12$, whose central term is an even number and, necessarily, the central term could not be odd, whose number of compositions are determined by $c_{12} = F_{12+1} = F_{13} = 233$.

Before finishing this section, we once again refer to Grimaldi (2012), which establishes a way to determine the number of palindromes in a composition. Indeed, considering the positive integer $n=12$, to determine the palindromes in the set of compositions $F_{13}=233$, we consider the cases: (i) if the central term of a composition is a plus sign '+' we consider it in the form of a 'reflection in the mirror'; examining the compositions of the integer $n=6$, such as in the case of

$$\underbrace{[2+2+2]+[2+2+2]}_{\text{central term}}$$

or

$$\underbrace{[1+1+1+1+1+1]+[1+1+1+1+1+1]}_{\text{central term}}$$

and which are equivalent to the amount of $n=6 \therefore F_7=13$ palindromes; (ii) If a number occurs as the central term, Grimaldi (2012) states that it must be even (that is, the digit '2' must occur), as in the previous example we wrote:

$$\underbrace{[1+1+1+1+1]+2+[1+1+1+1+1]}_{\text{central term}}$$

and one must consider, in this case, the compositions of the integer $n=5 \therefore c_5=F_{5+1}=F_6=8$ palindromes.

Finally, by an additive principle, when considering the whole set of palindromes, we write $F_6+F_7=F_8=21$ palindromes present in the composition of the positive integer $n=12$.

WAYS TO COMPLETE A BOARD AND SOME THEOREMS

In the previous section we pointed out some properties that, through combinatorial arguments, reveal properties intrinsically related to the Fibonacci Sequence. Preserving some of the previous arguments, we have Table 2, which allows relating the number of compositions of an integer n:

Table 2. Compositions c_n for a positive integer n from the digits '1' and '2'.

$f_1=1$	$f_2=2$	$f_3=3$	$f_4=5$	$f_5=8$	$f_6=13$		$f_7=21$	
1	11	111	1111	11111	111111	1221	1111111	12112
	2	12	112	1112	11112	1122	111112	21112
		21	121	1121	11121	2112	111121	12121
			211	1211	11211	2121	111211	21211
			22	2111	12111	1212	112111	12112
				122	21111	222	121111	12211
				212	2211		211111	11221
				221			11122	21112
							11212	1222
							2221	2122
								2212
$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$		$n=7$	

In the interest of providing greater rigor and increasing details in our discussion, we have established, from now on, the following definition:

Definition 1: (Board) The board is a formation of squares called squares, cells or positions. These positions are enumerated and this enumeration describes the position. Such a board will just be called *n*−board (Spreafico, 2014).

Next, we visualize the example of a 4–board. For its filling and possible configurations, the authors Benjamin and Quinn (2003) use only squares 1×1 (in the lighter color) and dominoes 1×2 (in the darkest color). Easily, if we wish to determine all possible tiling sets, including squares and dominoes, with the support of Figure 6 we can conclude that they are a total of $F_5=5$ possibilities (Figure 6, left). We could still choose just the set of tiles that have at least one domino (in the darkest color) 1×2 and, in this case, determine compositions of squares and dominoes. That is, we eliminate the first configuration only with the presence of squares 1×1. In Figure 7 we visualize the generalization of an *n*–board and then establish a relationship, in view of Theorem 1, with the Fibonacci sequence:

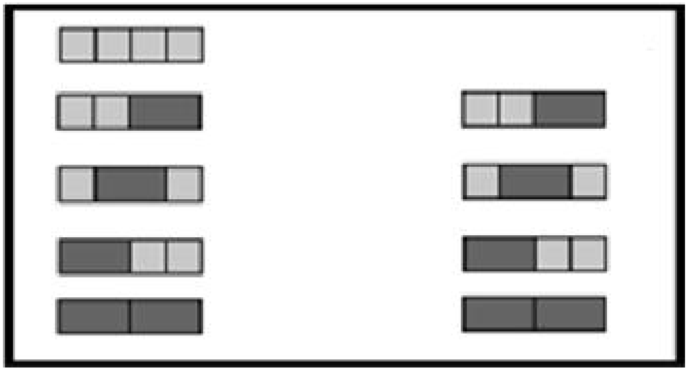

Figure 6. Benjamin and Quinn (2003) provide a particular representation via 4–Board.

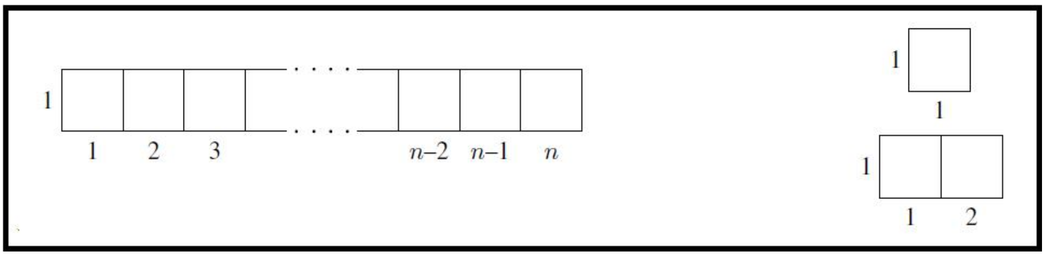

Figure 7. Benjamin and Quinn (2003) provide a representation via 'Board' related to the Fibonacci sequence.

Before proceeding, it is urgent to formalize certain heuristic and intuitive operations and arguments used just now, considering Theorem 1.

Theorem 1: The number of ways to cover a board $1\times n$ with squares 1×1 and dominoes 1×2 is equal to f_{n+1} (Spivey, 2019).

Demonstration: We can define as c_n the number of forms to cover a board $1\times n$ with squares 1×1 and dominoes 1×2. For a board of the type 1×1 we use only a square 1×1, i.e., we have $c_1 = 1 = F_2 = F_{1+1}$. For a board of the type 1×2, we have two possible configurations: with two squares or with one domino 1×2. In this case, note that $c_2 = 2 = F_3 = F_{2+1}$. Then, when we consider the number c_n as the number of ways to cover a board $1\times n$, there are only two possibilities, namely: (i) a set of partitions whose first piece is just a square 1×1; (ii) a set of partitions whose first piece is exactly a rectangle 1×2. if it occurs (i), that is, the first piece is a square, so the other positions $(n-1)$remaining must correspond to $c_{n-1} = F_n$.

With the same reasoning, if (ii) occurs, that is, the other positions $(n-2)$ remaining must correspond to $c_{n-2} = F_{n-1}$. To consider the set c_n as the total number of ways to cover a board $1\times n$, by an additive principle, we will add $c_n = (c_{n-1} + c_{n-2}) = F_n + F_{n-1} = F_{n+1},\ n \geq 1$.

In Figure 8, Grimaldi (2012) seeks to determine the amount of filling a board 2×3. The author notes that horizontal and vertical dominoes can be used (see Figure 7, right). The author explains that if we have a board 2×2 we have two ways of tiling: using two horizontal dominoes 1×2 or two vertical dominoes 2×1. If we consider a board 2×3 (Figure 8a), the author suggests decomposing the previous figure and counting the tiles for the case of the board 2×2, in which we have $q_2 = 2$ (Figure 8c).

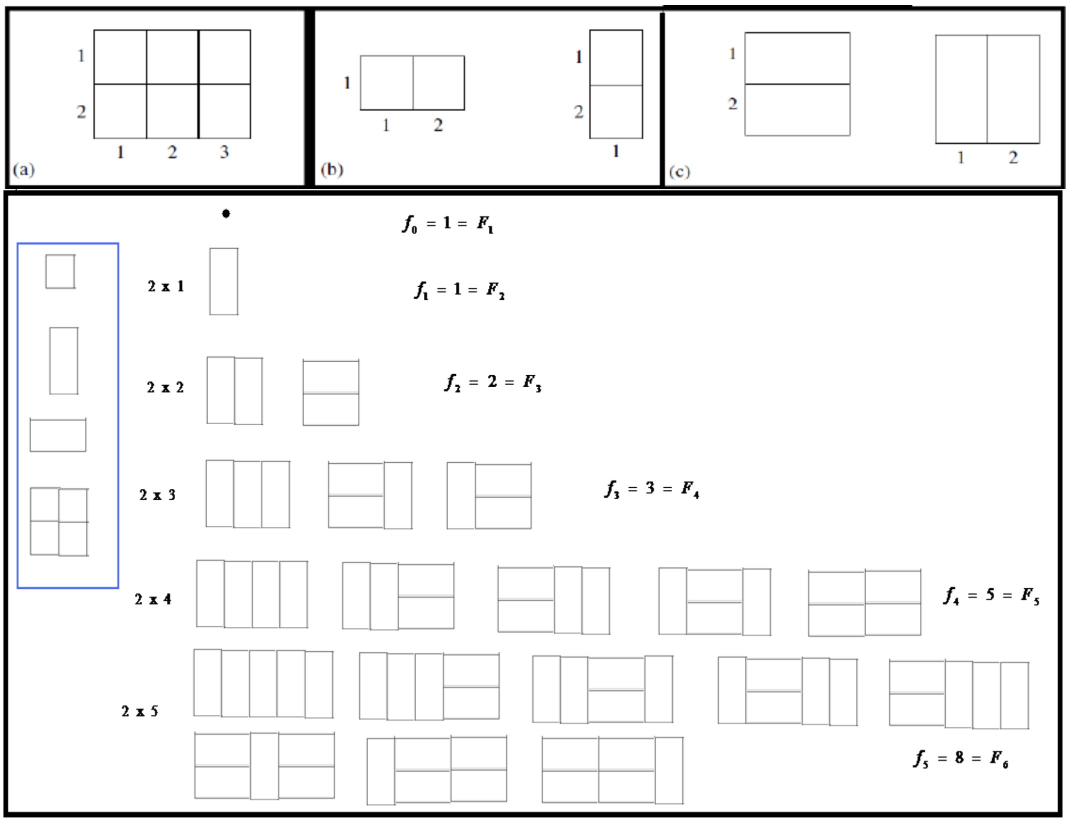

Figure 8. Grimaldi (2012) presents a board 2×3 and describes a relation with the sequence.

Theorem 2: The number of ways q_n to cover a board $2\times n$ with squares 1×1 and dominoes 1×2 is equal to $q_n = F_{n+1}$.

Demonstration: In the general case, for a board $2\times n$, we have two possibilities: (i) the first domino is vertical, so the remaining board will be of the type $2\times(n-1)$ and we will cover it of q_{n-1} distinct ways; (ii) by two horizontally juxtaposed squares, then the remaining board will be of the type $2\times(n-2)$ and we'll cover it for tiling for a total of q_{n-2} distinct ways. Considering both possibilities, by a combinatorial principle, Grimaldi (2012) establishes that $F_n = q_n = q_{n-1} + q_{n-2},\ n\geq 3$ and $q_1 = 1 = F_2,\ q_2 = 2 = F_3$.

COMBINATORY INTERPRETATION OF ELEMENTARY IDENTITIES

In the preceding sections, we used arguments and reasoning eminently of a combinatorial nature, aiming to show a character of arithmetic invariance of the recurrent sequence defined by $F_{n+1} = F_n + F_{n-1}$, with the initial values $F_0 = 0$ and $F_1 = 1$, whose combinatorial properties are usually neglected in History of Mathematics textbooks. Now, let us remember some identities that we found in the literature of this area of knowledge, such as:

$$\sum_{i=1}^{n} F_i = F_{n+2} - 1$$

which, according to Koshy (2001) was demonstrated by the French mathematician François Édouard Anatole Lucas in 1876, and similarly, the finite sums

$$\sum_{i=1}^{n} F_{2i-1} = F_{2n}$$

and

$$\sum_{i=1}^{n} F_{2i} = F_{2n+1}\ .$$

Grimaldi (2012) comments that other mathematicians, such as Giovanni Domenico Cassini (1625-1712) and Robert Simson (1687–1768) also found several ways to verify such combinatorial identities. The author also explains that in 1901, Eugen Neto (1846–1919) studied the set of compositions of a positive integer n, with a different method, except for the occurrence of the digit '1', as we can see in Table 3. The author suggests observing arithmetic relations and that, for these initial cases, we can establish the relation $e_n = F_{n-1}, n\geq 1$. On the other hand, Grimaldi (2012) adds that the determination of compositions e_n can be calculated from the recurrence $e_n = e_{n-1} + e_{n-2}, n\geq 3$, with $e_1 = 0, e_2 = 1$.

Table 3. Compositions of a positive integer, except for the digit '1'.

n	e_n	Compositions	n	e_n	Compositions
1	0	-	6	5	6, 2+4, 3+3, 4+2, 2+2+2
2	1	2	7	8	7, 2+5, 3+4, 4+3, 5+2, 2+3+2, 3+2+2, 2+2+3
3	1	3	8	13	8, 2+2+2+2, 2+2+4, 2+4+2, 4+4, 3+5, 5+3, 2+3+3, 2+3+2, 3+2+2
4	2	2+2, 4	9	21	3+3+3, 3+6, 6+3, 2+2+5, 2+5+2, 5+2+2, 4+5, 5+4, 2+2+5, 2+5+2, 5+2+2, 2+7, 7+2,
5	3	5, 2+3, 3+2	10	34	10, 2+2+2+2+2, 4+2+2+2, 2+4+2+2, 2+2+4+2, 2+2+2+4, 4+3+3, 3+4+3, 3+3+4, 6+4, 4+6, 2+8, 8+2, 4+4+2, 4+2+4, 2+4+4, 3+5+2, 3+2+5, 2+3+5, 2+2+7, 2+7+2, 7+2+2, 3+7, 7+3, 5+5, 2+3+2+3, 3+2+2+3, 2+3+2+3, 2+6+2, 6+2+2, 2+2+6, 2+5+3, 5+2+3.

Now, let's return to the set of palindromes that occur in the compositions of the integer n (excepting the digit '1'). When we consider the method proposed by Eugen Neto (1846–1919), we determine the number of compositions $e_n = F_{n-1}, n \geq 1$. To exemplify, let's take $n = 15$. Grimaldi (2012, p. 28) observes that, in the case where n is odd, “then the central term of the sum will always be an odd number”. For example, taking the particular composition:

$$2+2+2+\underbrace{3}_{\text{central term}}+2+2+2$$

The smallest central term to be used will be '3'. In this case, we consider the compositions corresponding to each side of the central term. For example, in the decomposition:

$$[2+2+2]+\underbrace{3}_{\text{central term}}+[2+2+2]$$

We consider $[2+2+2]$ and, taking $F_{6-1} = F_5 = 5 = e_6$ that correspond to the integer $n = 6$ (same to the left side). Therefore, there are $F_5 = 5$ palindromes in the compositions of $n = 15$, as central term equal to the digit ’3’. If the middle term is

$$[2+3]+\underbrace{5}_{\text{central term}}+[2+3],$$

We determine that $F_{5-1} = F_4 = 3 = e_5$ there are three compositions of the palindromic type, which are:

$$\underbrace{[3+2]+5+[3+2]}_{\text{central term}},\ \underbrace{[2+3]+5+[2+3]}_{\text{central term}},\ \underbrace{[5]+5+[5]}_{\text{central term}}$$

Consequently, the total amount of palindromes present in the integer decompositions $n=15$ (Table 4) occur, according to the above conditions and using the identity:

$$\sum_{i=1}^{n} F_i = F_{n+2} - 1 : F_5 + F_4 + F_3 + F_2 + F_1 + F_0 + 1 = \sum_{i=0}^{5} F_i + 1 = (F_7 - 1) + 1 = F_7 .$$

Before wrapping up this section, we examine a combinatorial interpretation for the identity $\sum_{i=1}^{n} F_i = F_{n+2} - 1$, supported by the arguments recorded by Benjamin and Quinn (2003). By Theorem 1, we know that the number of ways to cover a board $1\times n$ with squares 1×1 and dominoes 1×2 is equal to $c_n = F_{n+1}, n \geq 1$.

Table 4. Determination of palindromes by the Eugen Neto method.

Central term	Number of palindromes	Central term	Number of palindromes
3	$F_5 = 5 = e_6$	11	$F_1 = 1 = e_2$
5	$F_4 = 3 = e_5$	13	$F_0 = 0 = e_1$
7	$F_3 = 2 = e_4$	15	1
9	$F_2 = 1 = e_3$		

To verify the identity, we tried to answer the following question: How many tilings of a $(n+2)-\text{board}$ have at least one domino 1×2?

In Figure 8 we can immediately see that for a $(n+2)-\text{board}$, there are $f_{n+2} = F_{n+1}$ possible tilings, however a tiling will not contain any dominoes 1×2 (which in the figure is indicated in gray color). This is the case of tiling only with squares 1×1. It follows from this fact that the total quantity containing at least one domino will correspond to the number $f_{n+2} - 1 = F_{n+3} - 1$ and we exclude the possibility with squares only, similarly to what we did in Figure 6.

Now, by examining Figure 8, we begin to consider the existence of dominoes and the position of the respective domino 1×2. Preliminary, it may occur: i) The position on the domino in $(n+1, n+2)$ and, in this case, only $f_n = F_{n+1}$ tiling in a $n-\text{board}$ (having fixed the position $(n+1, n+2)$); ii) In the next step, the position on the domino can occur in $(n, n+1)$, in which there will be a square in the position $(n+2)$ and in this case there are only $f_{n-1} = F_n$ tiling in a $(n-1)-\text{board}$; (iii) Then the position on the domino can occur in $(n-1, n)$, in which there will be a square in the positions $(n+1)$ and $(n+2)$ and, in this case, only $f_{n-2} = F_{n-1}$ tilings in a $(n-2)-\text{board}$.

Continuing with the previous steps, step by step, we can identify in Figure 9 and observe that the last domino will be in the position $(1,2)$. So, there must be $f_0 = F_1$ tilings, as there will be a first piece (the domino) and all the rest are made up of squares 1×1:

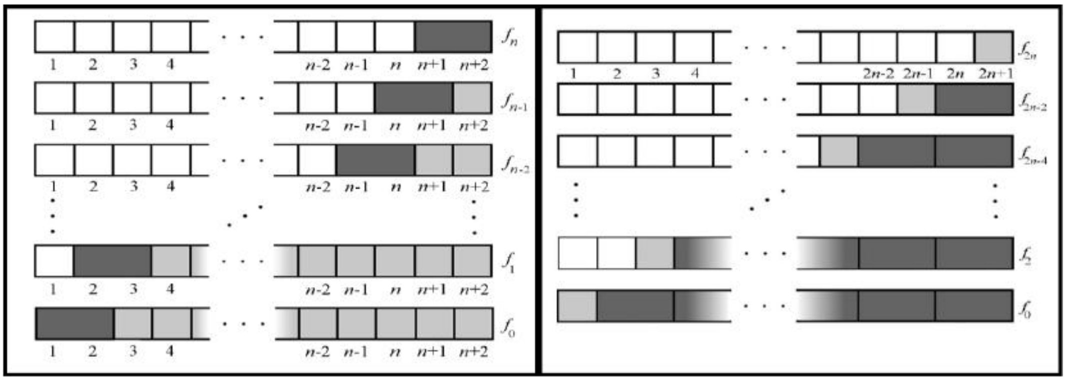

Figure 9. Representation via tiling for identities $\sum_{i=1}^{n} F_i = F_{n+2} - 1$ and $\sum_{i=1}^{n+1} F_i = F_{n+3} - 1$

Finally, summing all contributions involving tiling in steps F_{n+1}, F_n, F_{n-1}, F_{n-2}, ..., F_3, F_2, F_1, 0, it follows that:

$$F_{n+1} + F_n + F_{n-1} + F_{n-2} + \cdots + F_3 + F_2 + F_1 + F_0 = \left(\underset{\substack{N^\circ \text{ of the tilings on the (n+2)-board} \\ \text{excluding one tilings with the squares}}}{f_{n+2} - 1} \right) = \left(F_{n+3} - 1\right)$$

Note that the expressions $\sum_{i=1}^{n} F_i = F_{n+2} - 1$ and $\sum_{i=1}^{n+1} F_i = F_{n+3} - 1$ are exactly the same, except for the addition of one term. Still supported by Figure 7, Benjamin and Quinn (2003) provide an interpretation for the identity $\sum_{i=1}^{n} f_{2i} = f_{2n+1}$. For that, we consider a $(2n+1)-\text{board}$. Immediately, as there are an odd number of positions, therefore, every tiling must have at least one square (1×1). In Figure 7 (on the right), the authors Benjamin and Quinn (2003a; 2003b) consider the position of this square, which remains present in all tiling possibilities, since the length of the board is odd. Systematically, it should occur that: i) If the square 1×1 will be in the position $2n+1$ they must occur f_{2n} coverings or tiling forms in the $2n-\text{board}$ remaining; ii) If the square 1×1 will be in the position $2n-1$ they must occur f_{2n-2} coverings or tiling forms in the $(2n-2)-\text{board}$ remaining.

Repeating the previous arguments, we consider all contributions:

$$f_{2n} + f_{2n-2} + f_{2n-4} + \cdots + f_6 + f_4 + f_2 + f_0 = \left(\underset{N^\circ \text{ of the tilings in the (2n+1)-board}}{f_{2n+1}} \right)$$

In the previous sections we approached some elementary problems in combinatorics, whose arguments and representations used culminate, surprisingly, with the emergence of relations with the Fibonacci Sequence that, in certain specialized textbooks of History of Mathematics, tend to be neglected.

The topic discussed in this article has the potential to instigate future research on the applications of these sequences in combinatorial number theory and, possibly, other areas involving matrix algebra. In addition, we aim to broaden the study of other algebraic properties of the Fibonacci sequence in general, as well as the possibilities of applications of these sequences in teaching sessions, focused on the initial training of mathematics teachers.

FINAL CONSIDERATIONS

In the previous sections we addressed some elementary problems in Combinatorics whose arguments and representations used culminate, surprisingly, with the emergence of relations with the Fibonacci Sequence which, in certain specialized textbooks of the History of Mathematics are often not addressed and are rarely discussed in mathematics teacher training (De-Temple & Webb, 2014; Koshy, 2019; Spivey, 2019; Vorobiev, 2000).

In particular, the way to consider certain compositions of a positive integer n, with or without the presence of the digits '1' and '2' and, for example, we found that palindromes make it possible to relate and determine sets and subsets that, from a numerical point of view, correspond precisely to the numerical values that we indicate, in addition to the theorems involving tiling with squares and dominoes.

In our works, Alves (2017; 2022) we have indicated a non-static and evolutionary understanding of mathematical knowledge, from the birth stage of more primitive ideas, culminating in a specialized scenario, in which researchers and mathematicians from different countries express an interest in same math problem. In this way, elementary identities like $\sum_{i=1}^{n} f_i = f_{n+2} - 1$ and $\sum_{i=1}^{n+1} f_{2i} = f_{2n+1}$ that held the interest of professional mathematicians in the past (Stillwell, 2010) can be revisited, through combinatorial arguments and expressing heuristic properties for numerous compositions involving Fibonacci numbers (Hoggatt Jr & Lind, 1969).

Finally, the problems and approach we discussed in the preceding sections fall under what in Pure Mathematics we call “Combinatorics, which is often called Finite Mathematics, because it studies finite objects. But there are infinitely many finite objects, and it is sometimes convenient to reason about all the members of an infinite collection.” (Stillwell, 2010, p. 554). In these terms, Combinatorics stimulates the Mathematics teacher's understanding, from an elementary stage to the realization of a modern research scenario that confirms the vigor of research around Fibonacci numbers.

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