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Cultural Symbols Didactize Concreteness Fading in Basic Multiplication

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Abstract

The purpose of this study is to use cultural Adinkra artifacts to present "Concreteness Fading" in the basic multiplication of one-digit and one-digit numbers. Employing a quantitative approach, the researcher adopted a one-group pretest-posttest quasi-experimental design, and randomly selected 51 participants from 300 student teachers. Data collection involved two sets of tests, analyzed in two stages through task-based transcripts and paired-sample *t*-tests. The first stage analyzed the tasks the student teachers solved using "Concreteness Fading". The results revealed smooth and joyful navigations of the stages of Concreteness Fading using the Adinkra symbols. The second stage analyzed the performance of the student teachers with *t*-test statistics to show significant differences between the control and experimental groups. The results of one sample *t*-test and paired samples *t*-test showed that student teachers solved more problems correctly using Concreteness Fading than the conventional concrete manipulatives. Following the findings, we concluded that heavy use of only concrete objects and examples without abstracting can be detrimental to teaching mathematics. We, therefore, recommended that student teachers should always avoid rushing to symbols and symbolic manipulations of mathematics but rather align their methods, techniques, and strategies in the transition through the three stages of Concreteness Fading.

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INTRODUCTION

Jerome Brunner originally conceived Concreteness Fading in three developmental stages, and named them enactive, iconic, and symbolic (Bruner, 1974; Kim, 2020). The concept was later modified to Concrete, Representational, Abstract (CRA), or Concreteness Fading to refer to a three-step progression from a physical, diagram, and abstract states (Fyfe et al., 2014; Kokkonen & Schalk, 2021; Pearce & Orr, 2018). Through fading, student-teachers can "empty" the learned concepts of its specific sensory and perceptual properties, so they can grasp its formal, abstract properties (Pearce & Orr, 2018).

Research (e.g. Fyfe & Nathan, 2019; Kokkonen et al., 2022; Kokkonen & Schalk, 2021; McNeil & Fyfe, 2012; Pearce & Orr, 2018; Pickering, 2022; Suh et al., 2020) suggests that concrete representations get "faded" to yield more generalizable both during and after instruction. The abstract nature of the indigenous artifacts and their applications to the teaching and learning of mathematics require conscious fading.

Despite the long existence of "Concreteness Fading", it remains much swallowed! Moreover, it has evolved with many variants, making it difficult to know where to fit indigenous artifacts (Suh et al., 2020). Kokkonen et al. (2022) make more encouraging evidence for the effectiveness of concreteness fading. McNeil and Fyfe (2012) contend that no study has experimentally tested the effects of concrete-to-abstract representations. It is even inconceivable to embed artifacts like 'Adinkra' into "Concreteness Fading". Experimentations by Aduko and Armah (2022), and Donovan and Fyfe (2022) revealed significant differences between groups, suggesting potential benefits of concreteness fading. This study proffers local indigenous "Adinkra" artifacts that have always been glossed over. The virtues and values of these indigenous materials in mathematics learning cannot be over-sympathised (Clement Ayarebilla Ali, 2022).

An Overview of Adinkra Artefacts

Figure 1 shows nine of the Adinkra artifacts that were employed in this study. Each of these artifacts belongs to the different categories that the researcher has adapted and employed from Efiabevi (2013). The Akan word 'Adinkra' means "Farewell" (Kuwornu-Adjaottor et al., 2016). 'Adinkra' are traditional symbols that are primarily a usual translation of thought and ideas, expressing and symbolizing the values and beliefs of the people among whom they occur (Clement Ayarebilla Ali, 2021; Clement Ayarebilla Ali & Anderson, 2021). These symbols are embossed on textiles, pottery, stools, umbrella tops, linguist staff, logos, clothes, furniture, sculpture, earthenware pots, and many others (Babbitt et al., 2015; Boddy-Evans, 2020; Kuwornu-Adjaottor et al., 2016).

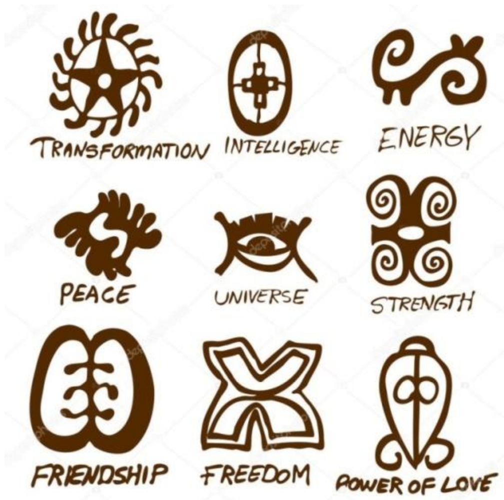


Figure 1. Indigenous Ghanaian "Adinkra" Symbols (Efiabevi, 2013)

Figure 1 shows a sample of nine 'Adinkra' symbols that the student teachers used to preview the concrete representations. They embossed them with interesting and thought-provoking English language interpretations. Some of these meanings are ‘friendship’, ‘power of love’, and ‘intelligence’. I selected the nine symbols purposely to motivate the student teachers to take up such positive values that can be mathematically interpreted. This encouraged them to actively participate in the experiments.

According to Okyere (2021), the Adinkra symbols were not originally used to teach mathematics. However, they still communicate strong and enviable mathematics ideas, values, and knowledge. The link of “Adinkra” symbols, and its immersed contributions to the teaching and learning of mathematics, in this context, multiplication of 1-digit by 2-digit numbers, was the major motivation. Adinkra symbols have mathematics digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9), operators (+, -, ×, ÷), and relaters (=, >, <, ≥, ≤) (Douglas et al., 2020). This brings to bear the context of mathematics in indigenous settings (Ali, 2021; Ali & Anderson, 2021; Kofi et al., 2021). Instead of the above literature, as a research question (RQ) in this study, how can “Adinkra” symbols help student teachers perform tasks in the three stages of multiplication of 1-digit and 2-digit numbers?

This research is so important for numerous reasons. Concreteness fading has variously been presented in other research works. The uniqueness of this study is the use of cultural artefacts. The new value of this study can be observed in the use of Adinkra symbols. These symbols abound in the environment. However, not many many people and mathematics educators link them to mathematics learning (Ali & Davis, 2016). By extension, the world of mathematics education could be expanded and extended to cultures and their artefacts. The symbols of mathematics are pure cultures of Arabic, Hindu, and European interplay. Mathematics learners, educators, and researchers of African origin may add this knowledge to their literature (Ali, 2021).

RESEARCH METHODS

In this section, the researcher presents the research design, experimentations, population and sample, data analysis instrumentations, and validity of the findings.

The Research Design

The researcher adopted a one-group pretest-posttest quasi-experimental design. In this design, groups of student teachers were classified based on their program of study, namely General Science, Social Studies and Humanities, Literacy and English Language, and Agricultural Science and Home Economics. First, the student teachers were given the “Adinkra” to learn and connect them to concrete materials. The design sought to delve deep into the principles of the model to link to the concept of multiplication of 1-digit by 2-digit numbers through the three stages (Kim, 2020).

The Experimentations

Two stages of experimentation were undertaken with 51 student teachers. These experimentations were based on two methods of teaching. The two methods of teaching were the conventional method as control and the “Concrete Fading” as experimental treatments respectively. In the control (CG), all the student teachers were tested before and after having explained the concept of the conventional method, after which a test was taken. The test before the teaching was called the 'pretest' and the test after the test was called the 'posttest' (Donovan & Fyfe, 2022).

In the experimental (EG), the students were first tested with the control (pretest). Then the student teachers were given a set of tasks in multiplications of 1-digit and 2-digit numbers. In the Concrete stage, real objects were used to teach the multiplications. The researcher did not move away from these concrete examples when explaining the concepts. In the representation stage, objects were drawn or taken snapshots. In the Abstract stage, no concrete examples were used and student teachers were taught using multiplication problems only (Kuepper-Tetzel, 2018).

In the experimental stage, the researcher started with concrete objects, then moved to a paper-based version that increased the abstractness of the representation (Suh et al., 2020), but still used the objects from before, and lastly concluded with numeric representations alone. The multiplication problems used during the first phase were relatively easy (e.g. 3×12) as compared to the ones student-teachers had to solve during the abstract stage (e.g. 32×25) (Kuepper-Tetzel, 2018). After they had completed the tasks, they were given a set of tasks in multiplication of 1-digit by 2-digit numbers called posttest. Then the worksheets were retrieved and analyzed based on the stages of the "Concreteness Fading" principle (Febriana et al., 2019).

Populations and Sample

Nearly 300 student teachers were offering Post Diploma in Basic Education programs at the University of Education, Winneba in Ghana (distance module). However, the researcher used the simple random sampling technique to select 51 student teachers who offered mathematics as their specialization. The 51 student teachers cut across genders, regions of Ghana, and knowledge of Ghanaian languages and culture. They also offered mathematics as their course for a professional teaching career and needed more methodologies and resources for improved learning outcomes. In the selection process, the researcher first took the index numbers of student teachers. Then a table of random numbers was generated to match the last digit of their index numbers. Any number that appeared on the table was selected (Ali, 2021).

Instrumentations

The researcher used two sets of instruments to collect the data. The first set of instruments was pretesting. In the pretesting, the researcher administered items based on the multiplication of one-digit by two-digit numbers via the conventional method of teaching. The second set of instruments was testing. In the testing, the researcher administered items based on "Concreteness Fading", having taken the student teachers through the indigenous 'Adinkra' symbols. The items were similar to those administered in the pretesting (Ali & Davis, 2018).

Data Analysis

The researcher used one sample *t*-test for the analysis to determine whether the sample mean was significantly greater or less than the hypothesized value of 0. The *t*-test was preceded by tasks on the Adinkra symbols. The data satisfied the *t*-test assumptions to determine whether there was statistical evidence between paired observations. The tool assumed that the data were measured in interval and ratio scales, the sample size was large, the data was homogenous, and the distribution was normal (Creswell & Creswell, 2018).

The measurements of the stages were taken at two different times with an intervention administered between the two time points, the measurements were taken under two different conditions and the measurements were taken from two halves or sides of the subject or experimental unit (Creswell & Creswell, 2018). This method helped the researcher to fashion out theories, policies, and practices that can incorporate indigenous artifacts into the Concreteness Fading.

Validity and Reliability

The researcher chose test-retest reliability to assess whether the performance in the conventional method yielded the same results as the performance in the "Concreteness Fading". Indeed, the Cronbach alpha coefficient yielded 0.78 (Bhattacherjee, 2023). In validity, the researcher assessed three main measures of validity, vis-à-vis criterion, construct, and content. This measured all the student teachers' knowledge in "Concreteness Fading".

Threats to Internal Validity

The researcher-controlled history (unrelated events such as fatigue that nearly influenced the performance), maturation (outcomes of the performance in the post-test that were nearly influenced by the pre-test), instrumentation (differences in times of the pre-test and post-test that nearly influenced performance) and testing (pre-test that nearly influenced the post-test). These were accomplished by manipulation, elimination, inclusion, statistical control, and randomization (Bhattacherjee, 2023).

Ethical Considerations

The researcher paid attention to voluntary participation, informed consent, anonymity, confidentiality, potential for harm, and results communication (Ali, 2022). Ethical protection aims to protect the rights of research participants, enhance research validity, and maintain scientific or academic integrity (Ali & Davis, 2018; Bhandari, 2020).

RESEARCH RESULTS

In answering this research question (RQ: How can "Adinkra" symbols help student teachers perform tasks in the three stages of multiplication of 1-digit and 2-digit numbers?), four tasks were used with "Adinkra" symbols. These included "Friendship", "Power of Love", "Strength" and "Intelligence".

Table 1. Descriptive statistics of participant scores

Group	Number	Minimum	Maximum	Mean	SD
CG (Pretest)	51	12	17	14.23	1.224
EG (Pottest)	51	17	20	16.22	1.032

In Table 1, the minimum value for the control and experimental stages was 12, and the maximum was 17. This gave a mean of 14.23(1.224) and 16.22 (1.032). It can be inferred that the mean values of the experimental stage were not only higher than the control but they were also more compact. The little variation was a good indication many participants had near-equal understanding after the interventions.

Table 2. Paired sampled *t*-test scores of participants' scores

Groups		Mean	SD	Confidence interval		*t*	*df*	Sig. (2-tailed)
				Lower	Upper			
Pair	CG vr EG	-3.12	1.65	-4.12	-2.12	-5.64	50	0.000

In Table 2, the significant value of the paired-sample statistics was less than 0.05. This means there was a difference between the control and experimental stages. Compared with Table 1, it can be concluded that the experimental stage produced a higher score resulting in the difference. This could be attributed to the cultural Adinkra artefacts that were used to better illuminate Concreteness Fading in Multiplication.

Task 1: The Results with "Adinkra" Symbol for "Friendship"

The student teachers brainstormed the issues around friendship and related the concept to learning. According to Dicken (2023), one may think a friendship will last forever but it is not uncommon for some friends to fade. Sometimes, a disagreement or falling out creates a gap between friends. Other times, commitments like work, distance, or family result in a friendship slowly fading away without animosity. Student teachers accepted the value of 'friendship' and believed that learning multiplication starts with own friendship. They applied the value during the group discussions in a friendly and cordial atmosphere. Concreteness Fading offered solutions for making these connections as in Figure 2.

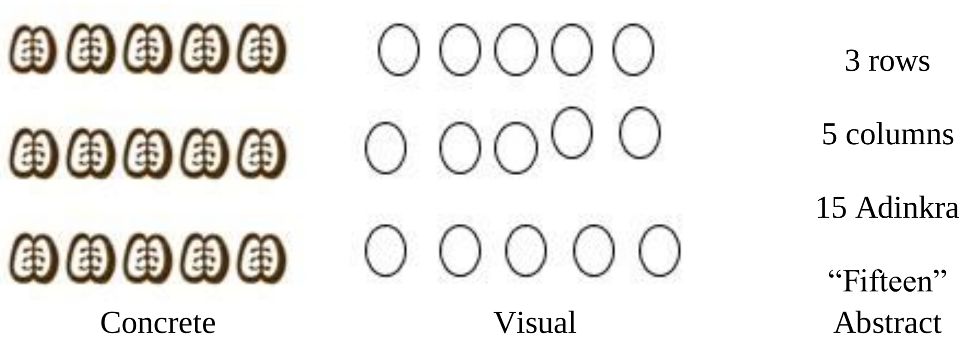
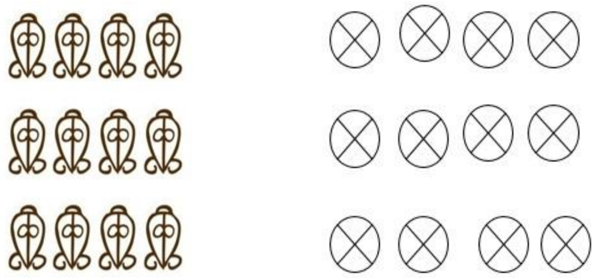


Figure 2. Concreteness fading without numbers (Pearce & Orr, 2018)

In Figure 2, Pearce and Orr (2018) started with these concrete manipulatives, progressed to drawing those representations, and finally, represented the mathematical thinking abstractly through symbolic notations. "Adinkra" are varied symbols and represent various mathematical contexts (Ali, 2019). The "Adinkra" symbols represent concepts or aphorisms and are used extensively in fabrics, pottery, logos, and advertising (Efiabevi, 2013). Even though one cannot directly link them to numbers and operations, they abound in rich affective attitudes (strength, forgiveness, faithfulness, friendship, and peace) and values (e.g. royalty, truth, courage, and understanding) enshrined in the new Ghanaian curricula of pre-tertiary education levels (Kofi et al., 2021; Ministry of Education, 2019).

Task 2: The Results with “Adinkra” Symbol for “Power of Love”

The power of love for mathematics has widely been discussed. For instance, Brown (2019) opines that the four ways to add love for mathematics are to commit to inspiring teaching, preach the value of mistakes, advocate for growth mindsets, and strengthen your mathematics skills. These four ways were adequately exemplified in carrying out the activities with the “Adinkra” symbol in the “Concrete Fading” as shown in Figure 3.



$$\begin{array}{r} 3 \\ \times 4 \\ 12 = (10+2) \end{array}$$

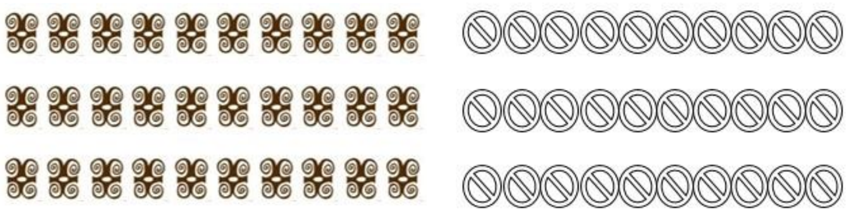
Concrete Visual Abstract

Figure 3. Linear Concreteness Fading without Numbers (Pearce & Orr, 2018)

Figure 3 shows how the conceptual understanding of “Love” continues to deepen through the use of the “Power of Love”. At this stage, student teachers continued sharing using visuals and gradually introducing symbolic notations. Different representations of concrete materials were required to consolidate learning. Moreover, since student teachers had a significant amount of time to inquire, investigate, and solve problems using both concrete and visual representations, they readily developed the ability to visualize different representations in their minds. It was more efficient to use symbolic notations and operations in multiplication rather than building concretely or drawing visually (Pearce & Orr, 2018).

Task 3: The Results with the “Adinkra” Symbol for “Strength”

Bowen (2021) has acknowledged a strength-based perspective of learning mathematics and advocated for lessons from a strengths-based perspective. This means focusing on what students already know, uncovering their strengths, and building on those strengths through instruction. This view is an excellent way to start using symbols from student teachers’ environment culture and customs and scaffold them to much higher heights (Kofi et al., 2021).



$$\begin{array}{l} 10 \\ \times 3 \\ 30\ (3\ \textit{tens}) \end{array}$$

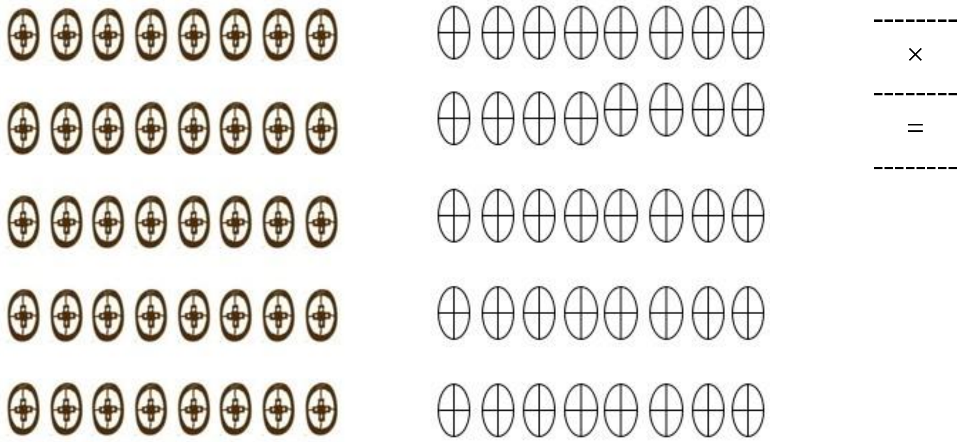
Concrete Visual Abstract

Figure 4. Concreteness fading with small numbers (Pearce & Orr, 2018)

Figure 4 shows how “Strength” propelled the student teachers to adequately prepare and enter into the abstract stage. As student teachers were shown how easy multiplication can be performed by having a conceptual understanding, they eventually jumped on the opportunity to multiply numbers without using manipulatives and representations. The more students linked their concrete and visual models to more abstract representations, the stronger their conceptual understanding supported any procedural approaches to progressive and effective algorithms in multiplication (Pearce & Orr, 2018).

Task 4: The Results with "Adinkra" Symbol for "Intelligence"

The fourth task was "Intelligence". Logsdon (2022) found that children with increased logical-mathematical intelligence are typically methodological and logical in thinking. Children may be adept at solving mathematics problems in their heads and drawn to logic puzzles and games. Mathematics "intelligence" exhibits children's abstract concepts, categorization, classification, patterns, problem-solving, and visual analysis (Bartolomei-Torres, 2022).



Concrete Visual Abstract

Figure 5. Concreteness fading with scaffolding activity (Pearce & Orr, 2018)

In Figure 5, student teachers have fully applied intelligence to transition and transfer knowledge into the abstract stage. In this stage, they scaffolded the concreteness fading models; they could easily conceptualize the movement without being assisted (Horn-Olivito & Martinovic, 2017; Kofi et al., 2021).

DISCUSSION

As originally conceived, it is a three-step progression that begins with enacting a physical instantiation of a concept, moves to an iconic depiction, and then fades into a more abstract representation of the same concept (Suh et al., 2020). It takes its source from Bruner's iconic, enactive, and symbolic stages (Bruner, 1974). This shows that learning from concrete materials that "fade" to abstract symbols benefits transfer, the progression from concrete to abstract is better than the reverse, learning from concrete materials is similar to learning from abstract symbols, and the benefits of "fading" extend to children with low and high prior knowledge (Fyfe et al., 2015).

Ching and Wu (2019) examined the effectiveness of various instructional strategies that aimed to enhance children's understanding of the inversion concept using 140 kindergarten pupils randomly assigned to each group of concrete-only, abstract-only, concreteness fading, abstract-to-concrete, and control. All the intervention (experimental) groups showed significantly greater progress than the control group in solving the inversion problems in the post-tests. In their findings, it was revealed that concrete representations were more effective than abstract representations. The superior benefits of concreteness fading appeared more prominent in the post-test scores for children with lower prior knowledge. The findings of Ching and Wu (2019) brought to light two key implications: (1) concrete representations should not be avoided in teaching mathematics, and (2) the order of the various representations is key for effective learning.

In particular, Pearce and Orr (2018) made significant and inspirational findings on the “Concreteness Fading” of the multiplication of numbers. Pearce and Orr (2018) discovered that using circular manipulatives like doughnuts was more concrete than drawing doughnuts or using symbols (numbers and operations). It was revealed that using concrete manipulatives was still more abstract than using the actual items in the quantity being measured.

On average, student teachers performed more problems correctly using “Concreteness Fading” as compared to the conventional methods. Kuepper-Tetzel (2018) discovered that heavy use of concrete objects and examples without abstracting from them can be detrimental to solving mathematics problems. “Concreteness Fading” can also always be done in different ways, namely providing concrete examples first, then substituting concrete with more abstract ones, and, finally, moving completely to an understanding of the abstract principles (Kuepper-Tetzel, 2018). This has completely answered and satisfied the domain of the study.

CONCLUSION

The results of Tables 1 and 2, and the transcripts on Figures 2 to 5 have shown that the student teachers used Adinkra Artefacts to solve the problems on Concreteness Fading The findings of the results on Tables 1 and 2 show that student teachers could readily navigate easily from the concrete into the abstract stage in the multiplication of 1-digit by 2-digit numbers. Particularly, it was evident that the student teachers had a lot of fun and play when using the “Adinkra” symbols.

The findings in Table 2 also show that the performance of student teachers improved in the “Concrete Fading” tasks as compared to the tasks involving the usual conventional methods of multiplication. Therefore, starting every mathematics lesson with concrete materials is nonnegotiable. The improvement was much more remarkable in the transition from visual to abstract than from concrete to visual. This big improvement was attributable to the “Concreteness Fading” principle and it is “Adinkra” that added more impetus to the tasks. This is contrary to the expectation that real concrete materials enhance learning than abstract ones. It is rather the value and virtue of the concrete material that matters more!

We, therefore, recommend that student teachers must always avoid rushing to symbols and symbolic manipulations of mathematics. It is not uncommon to observe many student teachers rushing towards using symbols for the multiplication of 1-digit by 2-digit numbers. This tends to veer the classroom into instrumental learning without making any deeper understanding and appreciation of the concepts therein.

We also recommend that student teachers endeavor to help their children develop a firm grasp of their indigenous artifacts like “Adinkra” in every mathematics domain. Even though the curriculum is emphatic on the use of local and indigenous materials that are easily accessible to pupils, it is not uncommon to hear many teachers complain about a lack of teaching and learning resources. Materials are readily available for addition and subtraction. However, resources to use for multiplication activities in school mathematics remain untapped.

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