Analyzing Errors Pattern in Mathematics Achievement among Senior Secondary School Students: A Case Study

Bright Ihechukwu Nwoke, Darlington Chibueze Duru*, Chioma Ahanotu, Chidiebere Precious Ifediba
Department of Mathematics, Alvan Ikoku Federal University of Education, Nigeria
*darlington.duru@alvanikoku.edu.ng

Article Info

Abstract
Identifying student math errors is crucial as they hinder skill development in problem-solving, logic, and decision-making, given math's abstract nature. This study investigates mathematical errors among senior secondary school students in Owerri Municipal Council, Imo State. A descriptive survey research design was employed. The sample size comprised 150 senior secondary students in class three (SS3), randomly selected from five out of ten public senior secondary schools. It addressed three research questions and one null hypothesis tested. Data collection utilized two expert-validated instruments (Mock Mathematics Essay Score Sheet [MMESS] and the Mathematics Error Identification and Classification Checklist [MEICC]) with substantial reliability. The analysis involved frequency, percentage, and Chi-square methods. The findings revealed that the errors committed by students included conceptual errors, computational errors, defective algorithms, and wrong operational errors. Wrong operational errors were most common among male students, whereas female students were more prone to computational errors. A significant difference in the errors committed by male and female students was observed. Math teachers must address errors in class, engaging students to prevent recurrence. Findings suggest tailored interventions for diverse learning needs, highlighting gender-sensitive teaching. Future studies may explore long-term error trends and intervention efficacy.

Keywords
Achievement; Errors pattern; Mathematics.

How to Cite:

INTRODUCTION
Mathematics is a fundamental discipline that involves the study of quantity, structure, and space. It deals with numbers (quantities and values) and their operations, ranging from fundamental processes such as calculation, computation, systematic problem-solving, etc. (Yadav, 2019). These processes are essential for understanding and applying mathematical concepts in various practical situations across different fields. Mathematics is not only an academic subject but also a vital tool used in diverse fields like science, engineering, technology, finance, and
healthcare (National Research Council, 2022). For example, it provides the framework for scientific models, powers algorithms in computers, helps predict market behavior, optimizes logistics, predicts weather patterns, and develops artificial intelligence. All these advancements take place within educational settings, where teaching and learning are essential for nurturing the understanding and application of mathematical principles.

In education, mathematics is considered a core subject taught from primary to secondary school, serving as a prerequisite for admission to higher education institutions (Federal Republic of Nigeria, 2013). It provides students with the necessary skills to analyze and solve complex problems, develop logical thinking abilities, and make informed decisions. A strong foundation in mathematics is vital for success in higher education and future careers, as it is often a prerequisite for fields such as science, technology, engineering, and mathematics (STEM). It is an essential subject that plays a pivotal role in students' academic success and future career prospects. By mastering mathematics, students gain confidence in their abilities to tackle challenging problems and develop a growth mindset, which is essential for lifelong learning and personal development.

A poor foundation in mathematics at the secondary school level can have significant implications for future achievement in the subject. Mathematics is a cumulative discipline, where concepts and skills build upon one another. Without a solid understanding of the foundational principles and fundamental concepts, students may struggle to grasp more advanced topics and apply mathematical reasoning effectively. A weak foundation can lead to difficulties in problem-solving, critical thinking, and logical reasoning, hindering students' ability to succeed in higher-level mathematics courses and limiting their academic and career opportunities that require mathematical proficiency (Wriston, 2015).

Despite the perceived importance of mathematics, there has been a lack of improvement in students' achievement in the subject at the final secondary school examination over the years, as observed and reported by WAEC Chief Examiners (West African Examinations Council, 2020). This indicates a pressing issue in mathematics education. Senior secondary school students face numerous challenges when learning mathematics, including difficulties in comprehending mathematical concepts, employing effective problem-solving strategies, and avoiding common errors. These errors include calculation mistakes, misconceptions, lack of attention to detail, misinterpretation of problem-solving prompts, etc. Such challenges significantly impact students' overall performance in mathematics and hinder their ability to apply mathematical knowledge and skills proficiently. Based on the report of the Chief Examiner of Mathematics (West African Examinations Council, 2020), it was observed that students commonly commit errors in specific areas of mathematics, including coordinate and circle geometry, trigonometry, three-set problems, word problems involving simultaneous equations, bearing and distance, and differential calculus. When students learn mathematics, they often make mistakes or errors in their calculations, reasoning, or problem-solving approaches. Identifying these errors is important because they can hinder the development of important skills needed for problem-solving, logical thinking, and informed decision-making.

The concept of an error is often employed with nuanced variations and is sometimes used interchangeably with related terms like misunderstanding, slip, and
mistake. According to Gardee and Brodie (2015, 2022), a distinction can be made between errors and slips. They propose that errors occur at a deeper conceptual level compared to slips, which are typically caused by carelessness or momentary lapses in attention. Errors, as described by Gardee and Brodie, are systematic in nature and stem from misconceptions or incorrect interpretations resulting from overgeneralizations of prior knowledge. These errors are rooted in fundamental misconceptions and can lead to persistent misunderstandings or flawed reasoning. On the other hand, slips are often characterized by minor, unintentional mistakes that occur due to lapses in attention, lack of focus, or carelessness. Slips are typically not associated with conceptual misunderstandings and may be more easily rectified or corrected once recognized.

The presence of errors in various fields, such as mathematics, measurement, data analysis, programming, or other domains, has significant implications for the reliability and accuracy of results (Pressman, 2016). Errors can arise from a variety of sources, including human factors, limitations of measurement instruments, approximations, or misconceptions. These errors can lead to flawed conclusions, inaccurate predictions, or compromised system performance. Mathematical errors refer to those pervasive errors that students make based on the difficulties they have experienced when dealing with mathematical problems (Herholdt & Sapire, 2014). In mathematics, an error refers to a deviation or mistake made during calculation, measurement, or the interpretation of mathematical concepts or results. Errors can provide opportunities for learning if the reasoning and underlying errors can be identified and explored as part of learning activities.

Various types of mathematics errors have been described in the research literature depending on the research purposes and theoretical approaches: visual-spatial errors, comprehension errors, transformation errors, relevance errors, fact errors, procedural errors, measurement errors, presentation errors, conceptual errors, wrong Operational errors, defective algorithm and computational errors (Rong & Mononen, 2022). In this study, the following four errors were used: conceptual errors, wrong operational errors, defective algorithms, and computational errors.

Conceptual Error occurs because the student has misunderstood the underlying concept or has used the wrong logic (Chamundeswari, 2014). Conceptual errors could be classified into four which include; incorrect use of formula, misinterpreting concepts, mistyping formula and not writing equations to answer questions. Computational errors happen when students understand the concept but make careless errors in computation. These are mistakes made when multiplying, dividing, adding or subtracting. The process was completed correctly and the student usually has a solid understanding of the concept, but somewhere along the way, they miscalculated. These may seem like more simple errors but they can really weaken the performance of students. Defective algorithm is when correct operation is applied but error in the procedure. This procedural error occurs when different procedures are used to answer the question and provide slightly different answers. The error within a procedure or steps that causes the result not to be as intended. Wrong operation errors refer to errors that occur when an incorrect operation is performed in solving problems. When operation performed is different from the expected one. Students, regardless of gender commit mathematics errors.
Gender, as defined by the World Health Organization (2019), refers to the societal and cultural expectations, roles, behaviors, and activities that are attributed to individuals based on their perceived identity as male or female. Gender disparities in mathematics learning and academic achievement have been a longstanding issue in education. Research has shown that there are significant differences between male and female students' mathematics achievement (Oribhabor, 2019). These findings suggest that gender differences in mathematics achievement may be linked to differences in error patterns. Analyzing error patterns and gender differences may help to develop effective interventions to improve mathematics learning outcomes for all students.

The occurrence of errors in learning mathematics among secondary school students in Nigeria has raised concerns due to the subject's crucial role in the country's growth and development. Various researchers have identified factors contributing to the commission of these errors, including poor teaching methods, ineffective communication skills, and a lack of technical know-how in improvisation (West African Examinations Council, 2020). Unfortunately, teachers have struggled to address these errors adequately, as there is a scarcity of studies investigating the specific types of errors committed by students, leading to weak performance in senior secondary school examinations.

Research Questions
In attempt to diagnose error patterns of students mathematics achievement among senior secondary schools in Owerri municipal of Imo state, the following research questions guided the study: (RQ1) What are the types of errors committed by senior secondary school students in the achievement of mathematics?; (RQ2) What type of error is mostly committed by male students? (RQ3) What type of error is mostly committed by female students?

Null Hypothesis
Null Hypothesis (H0): There is no significant difference in the error patterns in mathematics achievement between male and female senior secondary school students in Owerri Municipal Council, Imo State

RESEARCH METHODS
The study adopted a descriptive survey research design, which is characterized by the systematic collection, analysis, and interpretation of data to describe a phenomenon. In this case, the focus was on analyzing and establishing the errors made by senior secondary school students in mathematics within the context of Owerri Municipal Council, Imo State. This study was carried out in Owerri municipal L.G.A which came into existence in 1996. This city was found due to the British influence and colonization in the early 1900s, Owerri town was the headquarters for Owerri Division and later old Owerri Province. Also, when Imo State was created on the 3rd of February 1976, Owerri city was chosen as its capital. On the 15th of December 1996 Owerri city attained municipal status. Owerri Municipal has an urban setting with one autonomous community made up of 5 indigenous kindred (Owerre Nehi ise) vis: Umuororonjo, Amawom, Umuonyeche, Umuodu and Umuoyima, under the rulership of one paramount traditional ruler. Its
population is about 201,420 as at 2015 census. The other areas that make up Owerri Municipal include: Ikenegbu Layout, Shell Camp, Aladinma; Housing Estate, New Owerri as well as Trans-Egbu. Also, Owerri Municipal Council area shares geographical boarder with Owerri North and Owerri West Local Government Area. Most of the people in Owerri Municipal are traders, civil servants, politicians, artisans, academician farmers and different other professional field.

The study reviewed two theories: Thorndike's Trial and Error Theory, which suggests individuals attribute success to their abilities and failure to external factors, and Attribution Theory, which examines how individuals explain outcomes as either internal or external factors. The targeted population for the study comprised of only senior secondary class three (SS3) students in public schools in Owerri Municipal Council of Imo State, Nigeria. The total population of the SS3 students in Owerri Municipal Council is 2884. This formed the total population of the study. The sample was made up of 150 SS3 students in Imo State. A random sample of five (5) public secondary schools in Owerri Municipal using simple random sampling techniques (balloting without replacement). From each school, thirty (30) SS3 students’ mock scripts was randomly selected for the study which is 5.2 percentage of the population.

Two instruments were employed for data collection: the Mock Mathematics Essay Score Sheet (MMESS) and the Mathematics Error Identification and Classification Checklist (ME ICC). The test item (1-10) was the number of essay questions in mock examination. Mathematics Error Identification and Classification Checklist (MEICC) was used to identify the four types of errors 1 Conceptual Errors, 2 Computational Errors, 3 Defective Algorithm, and 4 Wrong Operational Errors. The instruments were validated by experts. The Cohen’s Kappa yielded coefficient of 0.78 for MEICC indicates substantial reliability. The researcher analyzed the errors using the drafted Mathematics Error Identification and Classification Checklist (MEICC). The data collected were analyzed using frequency, percentage and Chi-square. Decision rule: \( p\text{-value}<0.05=\alpha \), we reject the null hypothesis. If \( p\text{-value}>0.05=\alpha \), we accept null hypothesis.

**RESEARCH RESULTS**

The presentation of results followed the sequence of the research questions and hypothesis raised.

<table>
<thead>
<tr>
<th>Error types</th>
<th>Number of students</th>
<th>Frequency</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computational</td>
<td>150</td>
<td>411</td>
<td>36.86</td>
</tr>
<tr>
<td>Conceptual</td>
<td>150</td>
<td>203</td>
<td>18.21</td>
</tr>
<tr>
<td>Defective algorithm</td>
<td>150</td>
<td>190</td>
<td>17.04</td>
</tr>
<tr>
<td>Wrong Operational</td>
<td>150</td>
<td>311</td>
<td>27.89</td>
</tr>
<tr>
<td>Total</td>
<td>1115</td>
<td></td>
<td>100.00</td>
</tr>
</tbody>
</table>

Table 1 shows that among the errors made by students in learning mathematics, 36.86% were associated with computational issues, totaling 411 occurrences out of 1115. Conceptual errors accounted for 18.21% of the total errors, reflecting 203 instances out of 1115. Additionally, 17.04% of the errors were identified as
defective algorithm errors, amounting to 190 occurrences. Wrong operational procedures constituted 27.89% of the errors, with 311 instances out of 1115.

Table 2. Analysis of errors committed by male students

<table>
<thead>
<tr>
<th>Error types</th>
<th>Number of students</th>
<th>Frequency</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computational</td>
<td>64</td>
<td>133</td>
<td>31.00</td>
</tr>
<tr>
<td>Conceptual</td>
<td>64</td>
<td>66</td>
<td>15.38</td>
</tr>
<tr>
<td>Defective algorithm</td>
<td>64</td>
<td>80</td>
<td>18.65</td>
</tr>
<tr>
<td>Wrong Operational</td>
<td>64</td>
<td>150</td>
<td>34.97</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>429</td>
<td>100.00</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 reveals that computational errors, specifically issues related to calculations, were the most prevalent, accounting for 31.00% of their errors. This corresponds to 133 occurrences out of a total of 429 errors. Additionally, conceptual errors constituted 15.38% of the errors, with 66 instances, while defective algorithm errors were observed at 18.65%, totaling 80 instances. The highest percentage of errors, 34.97%, was attributed to wrong operational procedures, with 150 occurrences.

Table 3. Analysis of errors committed by female students

<table>
<thead>
<tr>
<th>Error types</th>
<th>Number of students</th>
<th>Frequency</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computational</td>
<td>86</td>
<td>278</td>
<td>40.53</td>
</tr>
<tr>
<td>Conceptual</td>
<td>86</td>
<td>137</td>
<td>19.97</td>
</tr>
<tr>
<td>Defective algorithm</td>
<td>86</td>
<td>110</td>
<td>16.03</td>
</tr>
<tr>
<td>Wrong Operational</td>
<td>86</td>
<td>161</td>
<td>23.47</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>686</td>
<td>100.00</td>
<td></td>
</tr>
</tbody>
</table>

Table 3 shows that among these errors, computational issues were the most prevalent, accounting for 40.53% of the total errors, with 278 occurrences out of 686. Conceptual errors constituted 19.97% of the errors, with 137 instances, while defective algorithm errors represented 16.03% of the total errors, totaling 110 occurrences. Additionally, wrong operational procedures comprised 23.47% of the errors, with 161 instances out of 686.

Table 4. Summary of Chi-square analysis

<table>
<thead>
<tr>
<th>Error Types</th>
<th>Gender</th>
<th>Male Observed</th>
<th>Male Expected</th>
<th>Female Observed</th>
<th>Female Expected</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computational</td>
<td>Gender</td>
<td>133</td>
<td>158.13</td>
<td>278</td>
<td>252.87</td>
<td>23.09</td>
</tr>
<tr>
<td>Conceptual</td>
<td></td>
<td>66</td>
<td>78.10</td>
<td>137</td>
<td>124.90</td>
<td></td>
</tr>
<tr>
<td>Defective algorithm</td>
<td></td>
<td>80</td>
<td>73.10</td>
<td>110</td>
<td>116.90</td>
<td></td>
</tr>
<tr>
<td>Wrong Operational</td>
<td></td>
<td>150</td>
<td>119.66</td>
<td>161</td>
<td>191.33</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td>429</td>
<td>429</td>
<td>686</td>
<td>686</td>
<td></td>
</tr>
</tbody>
</table>

Note: Degree of freedom $(r-1)(c-1)=3$; Chi-square table value at $\alpha=0.05$ is 7.815.

From the Chi-square analysis in Table 4, the statement of the hypothesis is rejected; implying that there is a significant difference in the errors committed by male and female secondary students in Owerri Municipal Council of Imo State.
This is because the Chi-square calculated is greater than the critical value at 0.05 alpha level.

**DISCUSSION**

From the results of the findings, shows that the major errors committed by senior secondary school students in learning mathematics are computational, conceptual, defective algorithm, and wrong operational errors. Computational errors were the most prevalent among senior secondary school students in Owerri Municipal Council, accounting for 36.86%, while 17.04% of the errors were identified as defective algorithm errors, indicating a lower occurrence. The findings corroborate with Omosewo and Akanbi (2013), who conducted research on the analysis of errors committed by physics students in secondary schools in Ilorin Metropolis, Nigeria. Both the study on senior secondary school students in mathematics and the research on physics students in secondary schools likely share similar educational contexts and challenges. Common issues such as curriculum structure, teaching methodologies, or educational resources may contribute to comparable error patterns across different subjects. These errors may be attributed to carelessness on the part of students. The study provides insights into error patterns in mathematics learning among secondary school students, informing evidence-based practices and policy decisions to improve teaching and learning outcomes.

The study's findings revealed that wrong operational errors were the most common among male students. These findings corresponded with the research conducted by Arhin and Hokor (2021), where they indicated that a substantial number of errors made by students occurred during the transforming, processing, and encoding stages. The challenges may arise from students' difficulties in accurately transforming mathematical problems into absolute results, leading to misinterpretations and misrepresentations in their solutions. This correspondence highlighted a shared theme between the errors identified in the present study and those emphasized by Arhin and Hokor (2021), emphasizing the need to address challenges related to the transformation and interpretation of mathematical problems to enhance overall understanding and accuracy in mathematical operations among students.

The study revealed that the most common error committed by female students is computational error. This implies that a significant portion of mistakes made by female students in the study pertains to inaccuracies or challenges in mathematical calculations. Computational errors may stem from challenges in mastering foundational arithmetic skills. If students have not fully grasped basic mathematical operations, it can result in inaccuracies during more complex calculations. Computational errors often arise due to a lack of attention to detail. Students might overlook or misinterpret specific numerical values or operational symbols, leading to miscalculations. Misunderstandings in the step-by-step procedures of mathematical problems can contribute to computational errors. Students may struggle with the correct sequence of operations or the application of mathematical rules.

There was a significant difference in the errors committed by male and female secondary students in Owerri Municipal Council of Imo State. The findings of the study revealed that the total frequency of errors committed by female students was
significantly higher than that of their male counterparts. This finding contradicted Omosewo and Akanbi’s (2013) research, where the total frequency of errors committed by male students was reported to be significantly higher than that of their female counterparts. This discrepancy in results emphasized the variability in academic performance and error patterns across different regions and contexts, highlighting the importance of considering local factors and nuances in educational research. It also prompted a deeper exploration into the specific factors that contributed to gender-based differences in error rates in mathematics education within the Owerri Municipal Council. The study reveals significant gender-based differences in error rates, with female students committing more errors overall compared to their male counterparts. This finding challenges conventional assumptions and highlights the need to consider gender as a factor in understanding and addressing error patterns in mathematics education.

CONCLUSION

The discussion highlights the major errors observed among senior secondary school students in mathematics learning in Owerri Municipal Council, Imo State, including computational, conceptual, defective algorithm, and wrong operational errors. While interviews were not conducted as part of the study, the analysis provides insights into potential root causes of these errors and their implications for education.

Based on the findings of the study, it was concluded that senior secondary school students predominantly commit major errors in computational, conceptual, defective algorithms, and wrong operational areas. The most common error among male students is the wrong operational error, whereas among female students, computational errors are most frequently observed. A significant difference in error patterns between male and female secondary students in Owerri Municipal Council of Imo State was also identified.

Recommendations Based on the findings of the study, the following recommendations are proposed. Firstly, adopt appropriate teaching methods. Teachers should employ suitable teaching methods, particularly for mathematical concepts requiring practical approaches. Practical demonstrations can aid students in visualizing and understanding the fundamental elements necessary for mastering complex concepts. Secondly, follow a sequential order in teaching. Teaching and learning should follow a sequential order, introducing mathematics concepts from simple to complex. Maintaining a logical progression is crucial to prevent students from feeling disconnected or overwhelmed in the learning process. Thirdly, provide feedback and follow-up. Mathematics teachers should actively engage in evaluating and providing feedback on errors made by students. Additionally, regular follow-ups after class or home activities are essential to reinforce learning and address any misconceptions. Fourthly, emphasize problem-solving skills. Teachers should expose students to problem-solving scenarios to enhance their problem-solving skills. Practical problem-solving experiences contribute to a deeper understanding of mathematical concepts. Fifthly, encourage cross-checking of work. Teachers should emphasize the importance of students cross-checking their work after examinations. This practice not only reinforces accuracy but also instils a habit of careful review, reducing the likelihood of errors. Sixthly, present topic content
precisely. To avoid confusion, teachers should present topic content in a concise and clear manner. Clarity in instruction contributes to better comprehension and application of mathematical concepts among students.

The study's findings have important implications for science education and curriculum planning in Nigeria. It highlights the need for teachers to address specific errors in computational, conceptual, algorithmic, and operational areas among secondary school students. The study provides students with valuable insights into common errors in mathematical operations, allowing them to reflect on their own approaches and take proactive measures to avoid these errors. Curriculum planners can use the study's feedback on students' weaknesses to make necessary adjustments to the curriculum content, ensuring it remains flexible and responsive to the needs of teachers and students. The government also plays a crucial role in improving learning standards and supporting initiatives to overcome identified errors in mathematics education.

Further research could explore these issues through triangulation with interviews to enhance the depth of analysis and inform targeted curriculum and instructional design strategies. Additionally, longitudinal trends in error patterns and assess the effectiveness of intervention strategies in improving mathematical proficiency. The study focuses on Owerri Municipal Council in Imo State to understand the factors specific to the region that may influence error patterns in mathematics learning, which is important for developing targeted interventions for local students.

ACKNOWLEDGMENT
The authors extend their heartfelt gratitude to all those who contributed to the research and writing of this work, making it worthy of broad dissemination within the academic community.

REFERENCES


