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Instructional Design: Teaching Algebraic Equations to Grade 8 Students with Involvement of Mathematical Reasoning in Cambridge IGCSE Curriculum

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Abstract

Mathematics tends to be a subject that is not favored due to a misunderstanding of the whole concept. The author believes that through involvement of mathematical reasoning skills and proper instructional strategies, students understanding can be transformed and students will be able to relearn the concept. The aim of this paper is to help teachers to recreate their lesson plans in a form of an instructional design with the involvement of mathematical reasoning, specifically in learning and relearning algebraic equations (constructivism), that can further diminish the anxiety and frustration students experience due to misconceptions in their algebraic comprehension. Through a combination of theories related to the teaching and learning of algebra in secondary school and frameworks regarding instructional strategies, the betterment of mathematical reasoning is expected to go along the relearning process, and the students will be equipped to move forward in learning mathematics. This instructional design will provide a variety of learning activities ideas, including a sample lesson plan along with other supporting documents for learning activities that teachers can use in the classroom.

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INTRODUCTION

In school settings, mathematics oftentimes, is seen by students as a set of formulas and computations that they have to memorize (Cafarella, 2014). There are a lot of people, be it students nowadays, or just adults who studied mathematics before, think that mathematics is not a source of satisfaction, but rather a starting point of frustration, discouragement, anxiety, and tend to think that mathematics is just a tiresome chore (Ignacio, Nieto, & Barona, 2006).

Indonesia’s low numeracy skills have been shown in various international testing. Based on TIMSS (Trends in International Mathematics and Science Study) survey in 2015, it shows that Indonesia is in the 45th rank out of 50 countries being tested, scoring only 397 points in compare to the average being 500 points. Reasoning, for example in algebra and geometry, becomes the substance in the test.

Algebraic thinking or reasoning refers to the ability of forming generalizations from experiences with number and various forms of computation, formalizing these ideas with the use of a meaningful symbol system, and exploring the concepts of pattern and functions (Van de Walle, Karp, & Bay-Williams, 2013). Algebraic comprehension is deemed to be important as it underpins all mathematical thinking, allowing for students to explore the structure of mathematics (Ontario Ministry of Education, 2013; Yuni et al., 2021).

However, the teaching and learning of algebra is associated to various difficulties and challenges. Students often acquire a formal and routinized method, where they are bound to make errors while executing said algebraic procedures (Kontorovich, 2020). Due to the following of steps and absence of mathematical reasoning in students, they are unable to trace or correct their errors regarding algebraic problems (Wagner, 1983) Students who are able to perform routine questions at a formal level correctly often reflect a limited recognition of the algebraic ideas, and hence, are lost when the problem situation is slightly changed. There is a lack of mathematical flexibility to adapt to algebraic problem-solving procedure, unless students are able to refer to the more informal and natural method (Küchemann, 1981)

This brought light to mathematical misconceptions. Salah satu jenisnya adalah miskonsepsi, yaitu kesalahan pada sistem belief dikarenakan penalaran, intuition, culture, life experience or something else (Parwati & Suharta, 2020). Sarwadi and Shahrill (2014) researched on mathematical errors and misconceptions to have found that students' misconceptions contribute a major impact towards students' progress and achievement in mathematics. Hence, eradication of mathematical misconceptions can further improve students' scores in mathematics. In order to do so, teachers should be able to deliver materials to students in a way that students can understand, that students will be able to perceive mathematics in a new light. This is where mathematical reasoning comes in. Mathematical reasoning allows students to go beyond the routine use of procedures, and move towards learning concepts and properties as being logical, interrelated and coherent aspects of mathematics (Mata-Pereira & Ponte, 2017).

According to ISC Research in 2017, Indonesia has the largest number of international schools in Southeast Asia, placing Indonesia as the 10th position globally, with 192 international schools (Partami, Padmadewi, & Artini, 2019). Thus, it may come as a surprise that in Indonesia, there is a limited source for teaching mathematics in international schools. Most of the times, teacher relies heavily on curriculum-based text books, whereas instructions for the students are oftentimes not specified (Haggarty & Pepin, 2002) There is a need to break language barriers in such a diverse country, yet no solution has been addressed. Therefore, it is of the utmost importance that an instructional design is to be made: to cater the needs of teachers in the growing number of international schools in Indonesia.

RESEARCH METHODS

The methodology in this paper refers to the instructional design construction methodology, whereas it reflects a specific take of the ADDIE model of instructional design (Kurt, 2017) along with the logic of Backward Design (Wiggins & McTighe, 2005) The figure below (Figure 1) refers to the chart of the construction of the instructional design.

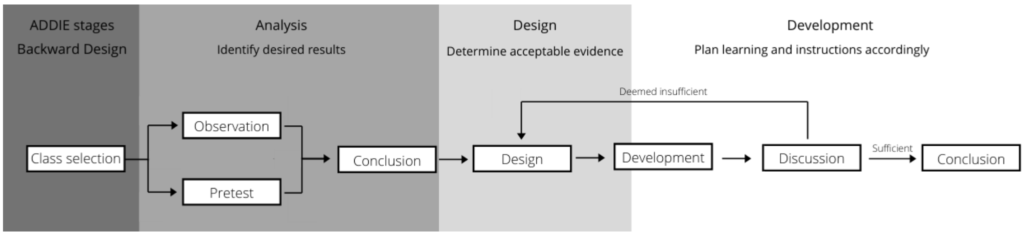

Figure 1: Construction of Instructional Design Based on ADDIE Model and The Logic of Backward Design

Going into details, this section will cover the steps in which the construction of the instructional design took place, according to the chart above (Figure 1) Before any construction of instructions was done, the author undergo class selection, which will be the basis of the analysis of needs, which is the first stage in the ADDIE model (Kurt, 2017), alongside with the first step in the logic of Backward Design (Wiggins & McTighe, 2005) This includes an observation and pretest, which serves as a form of analysis and not intended to be done as a proper research, as its sole purpose is to analyze the needs of the classroom for the formation of the instructional design; to identify the gaps in teaching and learning in the classroom, and to identify any misconceptions present in the students that may be inflicted in future teaching and learning activities.

The conclusion that can be made through the analysis of needs will be the grounds of the second stage in the ADDIE model, which is design. More specifically, it happens concurrently with the second step of the logic of Backward Design, which is to determine acceptable evidence. The construction of the instructional design starts from the literature review and selection of various frameworks that can support the instructional design, including the evaluation plan, which shows the complementary aspects of both ADDIE model and the logic of Backward Design. More precisely, the design will be based on the algebra content provided in the Cambridge Mathematics IGCSE Syllabus 2020-2022 and with involvement of mathematical reasoning through NCTM Reasoning Standards for grade 6-8.

Afterwards, it is followed by the actual construction of each part of the instructional design, whereas it is considered to be the third stage in the ADDIE model, which is development, and third step in the logic of Backward Design which is to plan learning and instructions accordingly. This means to start creating lesson plans that will make up the instructional design. The construction of the instructional design is not limited to the leson plans, but to the worksheets, presentations, and any additional teaching and learning support that will be utilized in the classroom as the instructional design is implemented. This part will utilize the logic of Backward Design from Understanding by Design (Wiggins & McTighe, 2005) whereas it will be the structure of the lesson plans, and the constructivist teaching steps, which will be utilized as the order of the learning activities that take place in the classroom.

The development stage is the stopping point of the instructional design, as further implementation and evaluation is going to be done under a specified context coming from each school. Following the development, it is then discussed and revised – going back and forth between design and development until it is deemed sufficient. Once it is deemed sufficient, conclusions regarding the limitations, strengths and weaknesses of the instructional design will be further elaborated based on the result of the instructional design constructed.

RESULT AND DISCUSSION

This section will further elaborate the actions taken in the effectuation of the ADDIE model that refers to the previously attached chart. This section will be divided into four subsections, each stand for each process and stage in the chart; analysis, design, development, evaluation.

Analysis

The analysis of needs in the ADDIE model of instructional design intends to measure the students' capability in algebra to find the underlying problems that exists in the classroom. This information will further guide the development of the instructional design. According to Brodie (2010), we can obtain traces of mathematical reasoning from classroom work and interactions. Hence to properly assess and identify the problem, the author conducted an observation in a class of 17 students for one week (7×40 minutes), and then gave the students a test of the algebraic topic that has been previously taught and tested until finished, which was translating general statements into mathematical statements (algebraic expressions.) Students are instructed to answer as best as they could, and if they cannot answer, they just need to explain why. Figure 2 shows the problem identification test questions.

Instruction: Translate the following statements into mathematical statements!

1 The product of two consecutive integers is 342.

2 The sum of two consecutive positive numbers is 1105.

3 A girl's age in 12 years time is the square of her present age.

4 The product of two consecutive integers is 380.

5 The sum of three times a number and two less than four times that same number is 61.

6 There are two consecutive positive odd numbers. The sum of the squares of the larger number and 4 times the smaller number is 349.

Figure 2. Problem Identification Questions

Corresponding to the proposition that mathematical communication is connected to mathematical reasoning (Kaur & Toh, 2012), and that involvement of mathematical reasoning will be part of the instructional design, the author relates students' ability in algebraic equations to mathematical communication. Hence, the questions of the problem identification refer to one type of question on algebraic comprehension that relates to mathematical communication; to translate statements into algebraic expressions (Molina et al., 2017) It is worth noting that the students are fluent in spoken English, whereas they earn their mathematical knowledge in the English language. The ethnic composition of the classroom consists of 60%

Indonesian students, while the rest are of various ethnicities. Thus, there should be no problem for students in interpreting the questions due to the language factor.

Problem Identification Result

The result shows that none of the students capable on answering all six questions. There are only 5 students capable of answering 5 out of 6 questions, and 5 students who are incapable of answering any of the questions. Table 1 and Figure 3 describe the distribution of the data based on the scores:

Table 1. Students' score on Problem Identification Sheet

Score Range	Number of students	Percentage
0-25	9	56%
26-50	2	13%
51-75	0	0%
76-100	5	31%
Total	16	100%

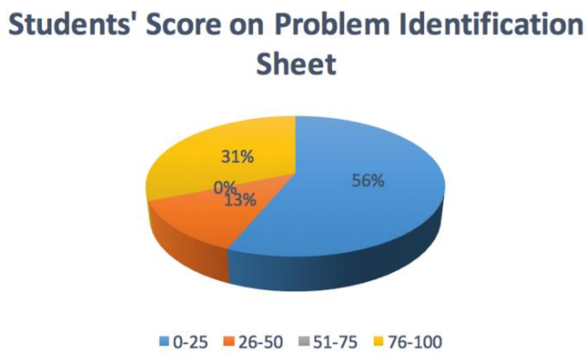

Figure 3. Students' score on the Problem Identification Sheet

The scoring system is made to be simplistic, students who are unable to reflect any form of mathematical thinking or to answer in any way are given 0, meanwhile an incomplete answer that reflects mathematical thinking is scored half. The full 1 score is given to students who are able to answer the question correctly. The score distribution is as follows; the majority of the students (56%) are unable to answer any of the questions, followed by the second majority of the students (31%) being able to answer most of the questions scoring 76-100. Additionally, a minority of the students are able to score 26-50. Looking at the score distribution, it can be inferred that most of the students do not understand how to answer the questions, considering the majority of the students were unable to answer any question. On the other hand, most of the students have problems in making mathematical models, as part of reasoning and generalization.

Observation and Reflection

The classroom observation is done by the author’s presence in the classroom. The author floated around the classroom while assisting the teachers with helping the students with their worksheets, talking with them as well to find the dynamic of the classroom. Additionally, the author conducted a reflection at the end of each session to ask how the students feel towards the class session each day.

According to classroom observation, teachers tend to only focus on the high-achieving students to confirm formative assessments; in-process evaluations relying on only a number of students instead of the whole class, making it easier for other students to be left behind, which suggests the score distribution. The students tend to be disengaged from the classroom and prefer to just stand by and idle. When it comes to structure, students, especially low achieving students, tend to be left behind in terms of their mathematical knowledge. For instance, in class, when discussing algebraic expressions, the observation result discovered a trend that there are students who do not know how to do basic arithmetic, how a mathematical expression is formed, or how algebraic manipulation work. Thorpe (2018) states that this indication shows how students are lacking in reasoning.

Through the reflection session, it is also observed that students are unfamiliar with mathematical terms, which is a problem, considering the teacher converse using mathematical terms. However, it is worth noting that the students came from an international school background where they converse in English, and they are not familiar with mathematical terms in Indonesian, hence, language barrier is not the issue. The students also express frustration towards mathematics, and they also complained about their classes. Hence, the problem revolves around students' comprehension in mathematical terms as well, as the students will not be able to understand the material being delivered if they are not familiar of the terms being used in the classroom, which includes the ability to make generalizations. If teachers are unaware of its presence, and are not in the habit of getting students to work at expressing their own generalizations, then mathematical thinking is not taking place (Zazkis, Liljedahl, & Chernoff, 2008).

After the completion of the problem identification test and each observation session, the author sat down and discuss the results with the mathematics teacher that is in charge of the class. The teacher stated that the class' performance by and large, also overwhelmed the teacher, as the teacher never had prior experiences with such low achievement in classroom performance (e.g. inability to do basic arithmetic or following instructions.) The teacher referred to the class as a result of excessive leniency on their mathematical performance. However, this is mere observation of the teacher, and it will be a part of the consideration in the construction of the instructional design.

According to NCTM's Reasoning Standard (NCTM, 2008), students who fail to understand and make sense of mathematical ideas and instead resort to rote learning, experience continued failure and withdraw from mathematics learning (Battista, 2017). This is a main problem that circles around the classroom; during the observation, students sent negative remarks related to mathematics and how they want to give up because mathematics cannot be understood. This may offer explanation to the students' behavior in the classroom. The author sees this as an opportunity to familiarize students with the mathematical concepts at hand, until they manage to be comfortable in making their own generalizations.

All in all, it can be concluded that the gap between the students' comprehension and capability is the unfamiliarity with mathematical terms that are commonly used in secondary school, the lack of overall mathematical comprehension (not knowing how to perform basic arithmetic, not knowing the differences between algebraic equations and expressions, etc.), and inability to reason nor identify the mathematical structure in questions. Hence, the instructional design should address the issues that act as a gap in the aforementioned grade 8 students' learning.

Design

The design stage is the second stage in the ADDIE model of instructional design which goes hand in hand with the second step in the logic of Backward Design (Wiggins & McTighe, 2005) The design stage will be based on the algebra content provided in the Cambridge Mathematics IGCSE Syllabus 2020-2022 along with NCTM Reasoning Standards for grade 6-8.

In Cambridge curriculum, students are introduced to algebra as early as their elementary school years. However, since it has been discovered that students in the problem identification struggle with the basics of algebra, the author give credence to a reintroduction to algebra as an attempt to familiarize students with the algebraic content that they have covered. This process leads students to communicate between themselves and their knowledge (Musanti, Celedón-Pattichis, & Marshall, 2009). Hence, a reinstitution of algebraic comprehension is done to straighten misconceptions that students may have regarding algebraic equations. This step may not be necessary when the context of the classroom differs and students do not face the same issues.

Furthermore, the reinstitution of algebraic comprehension will several aforementioned learning objectives (Cambridge International Examinations, 2016), which then will be implicated with the reasoning standards (NCTM, 2008) to form multiple activity designs that might support the learning activities in the respective content and reasoning standards. Table 2 shows the general recommendation of the practical activity design in reference to the IGCSE content section and NCTM reasoning standards. The design of this activity will be designed in the lesson plan.

As for the lesson plans, it will be made according to the structure suggested by Wiggins and McTighe (2005) alongside with the aforementioned constructivists' teaching steps that is part of the learning activities. The structure will include as follows. First, learning objectives; made using the Principles of Creating Learning Objectives in order to maximize the incorporation of reasoning in the lessons, the learning objectives will be closely knitted to the reasoning standard coverage. Second, reasoning standard coverage; incorporated in each part of the lessons to act as a reference. Reasoning standards are not hierarchical and are embedded in all parts of the lesson plans. This part of the structure is intended to also encourage teachers to take into their own initiatives to improve parts of the lesson plan. Third, resources; to help teachers prepare for the class with the aid and support of external resources, such as PowerPoint presentation, handouts, videos, index cards, small white boards, etc. Fifth, learning activities; to elaborate the activities that will take place in the classroom. Sixth, indicators and guidance; made as an attempt of formative assessment in the classroom, while giving teachers the freedom to adjust parts of the lesson according to the students and classroom environment.

Table 2: Activity Design According to IGCSE Content Sections and NCTM Reasoning Standards

IGCSE Content Section	NCTM Reasoning Standards	ActivityDesign
Reinstitution of algebraic comprehension 1. Use letters to express generalized numbers and express basic arithmetic processes algebraically. 2. Substitute numbers for words and letters in formulas. 3. Construct simple expressions and set up simple equations. 4. Manipulate directed numbers, use brackets, and extract common factors. 5. Derive and solve simple linear equations in one unknown.	Recognize reasoning and proof as fundamental aspects of mathematics.	Through the betterment of understanding the concept of algebra, students will recognize the importance of reasoning, it being the fundamental aspect of learning mathematics (Ramdani, 2011)
	Make and investigate mathematical conjectures.	The activities of making conjectures can be done in the classroom through discussions and provoking/scaffolding questions in worksheets and activities. When students are able to communicate clearly in mathematics, as they better their comprehension, students are able to make conjectures which then would further their understandings and make it easier for them to structure their thinking.
	Develop and evaluate mathematical arguments.	Following students learning on how to make conjectures, students are expected to be able to make arguments of their conjectures. This will help them intrinsically, as defending (arguing) to one's own statement is natural, and they will also then practice how to communicate their reasoning, making it easier for teachers to understand their train of thoughts. This takes place in discussions within group mates, which emphasizes the importance of group work (Prideaux, 2007)
	Select and use various types of reasoning and methods of proof.	Although the types of reasoning and methods of proof are limited due to the application to 8th grade students, students are at least expected to know how to reason and be aware that there is more than one way of thinking to achieve an answer to a question (NCTM, 2008) by knowing their own reasoning, the betterment of algebraic comprehension will be more complete.

Development

The development of the instructional design starts with the consideration of the learning objectives and the reasoning standards for the learning activities in the lesson plans. The development stage is also the stopping point of the construction of the instructional design as aforementioned. The product of the instructional design will include lesson plans, worksheets (opening activities and assessments), learning aids, and presentations. Figure 4 shows a sample of the lesson plan structure.

<table>
<tr><th colspan="6">Instructional Design: Reinstitution of Algebraic Comprehension</th></tr>
<tr><td>Year:</td><td>8</td><td>Unit:</td><td colspan="2">Algebra Review 1</td><td>Time:</td></tr>
<tr><td colspan="2">Learning Objectives:
1. Use letters to express generalized numbers and express basic arithmetic processes algebraically.
2. Substitute numbers for words and letters in formulas.
3. Construct simple expressions and set up simple equations.</td><td colspan="2">Reasoning Standards:
- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments.
- Select and use various types of reasoning and methods of proof.</td><td colspan="2">Resources:
- PowerPoint presentation
- Opening activity sheets
- Scissor & glue
- Algebra tiles paper (to cut)
- Algebra tiles worksheet</td></tr>
<tr><th>Indicators</th><th>Guidance</th><th colspan="2">Learning Activities</th><th colspan="2">Reasoning Standard Coverage</th></tr>
<tr><td>Students' attention is directed towards the teacher and students are willing to answer or ask follow-up questions.</td><td>Q+A</td><td colspan="2">Orientation: learning objectives, introduction to the importance of algebra.
1. Today, we are going to review algebra and learn the following learning objectives. (Slide 1&2.)
2. We all know how to answer basic mathematics questions. But in real life, we don't always know everything. There's always unknown values here and there that we need to</td><td colspan="2">Recognize reasoning and proof as fundamental aspects of mathematics.</td></tr>
</table>

Figure 4: Sample of lesson plan structure

However, the worksheets will be discussed here to elaborate on the reasoning aspects of the activities that will be present in the classroom. One of the activities involved in the classroom is the presence of an opening activity that is expected to trigger students' critical thinking and reason along with the questions in the worksheet. Figure 5 shows the opening activity titled "Making Homemade Orange Juice" where the activity will act to bridge students' prior knowledge to algebra through a familiar activity.

The questions present in the opening activity worksheet are not exactly algebra, but yet it involves algebraic thinking. This is to foster the previous gap that is present in the students, whereas prior knowledge that are considered to be the requisites for algebra are to emerge in students, such as ratio and proportions. For instance, question 1A attempts to trigger students into forming an informal algebraic expression/equation, to bridge students' thinking in everyday life to mathematical thinking. Meanwhile question 1B attempts to make students reason between the differences in variables, as students think about what makes them similar and different. Through scaffolding questions such as question 1B, students are expected to make conjectures of what they think is right or wrong in mathematics.

Question 1C attempts to formalize students' thinking of the recipe in the form of an algebraic equation, giving students the freedom to choose their own variables to represent each of the parts in the recipe. The questions present in the opening activity are not limited to those shown in Figure 5. This opening activity also presents an opportunity for the teachers to bring the activity into real life, whereas students can bring the ingredients and they can make the orange juice whilst discovering their prior knowledge aspects as they start to renew their algebraic thinking.

It is worth noting that the activities provided are not limited to the ones elaborated in the form of worksheets. There are various ways of assessing students and provoking their algebraic thinking. For instance, some activities may be made from the evaluation plan of the instructional design (formative and summative assessments).

Opening Activity 1: Making Homemade Orange Juice

I. **Answer the following questions!**

1. You want to make orange juice for your friends who are coming over on the weekend. The picture above is the recipe for orange juice.
   a) According to the recipe, what do you need to make a glass of orange juice?
   b) Is orange juice and oranges the same thing? How is it the same/different?
   c) Let *y* be orange juice. Form an algebraic equation of the ingredients in the recipe. (You can use any variable you want.)

Figure 5: Opening Activity Worksheet

Evaluation

The evaluation plan is the last step in the ADDIE model for instructional designs. As the implementation stage is not conducted, the evaluation stage is not conducted as well, although planned. The evaluation will cover specifically on the learning evaluation that the students undergo during the implementation of the instructional design.

Table 3. Evaluation plan and implementation

Evaluation level	Measurement activities	Potential Implementation
Level 1 – Reaction	▪ Write, Pair, and Share: pose a problem/question related to the learning objective being taught to the classroom for students to answer on their own. Let them switch with their seatmates and let them give inputs to each others' answers. Float and check around the class, make checkmarks on a personal clipboard as assessments for each student. The notes the students make will be the data that the teacher can use to observe and assess reasoning standards coverage, especially in making and investigating mathematical conjecture. ▪ Fist to Five: for a very brief formative assessment, teachers may use this strategy every 15-30 minutes of their class to check what students are feeling towards the lesson at hand. Ask students to raise their hands, a fist represents 0, and they can put up one to five fingers to represents their understanding and how they are feeling to the topics being learned.	All lessons in the lesson plans; the activities can be done in the form of worksheets to be shared with students' seatmates (or any form of pairs that is appointed by the teacher.) All lessons in the lesson plans; fist to five is incorporated almost in every section to check the students' progress. If it is deemed as necessary, the teacher can implement
Level 2 – Learning	▪ Round Robin: students form groups and teacher provide a question for them to solve. Students form a line and taking turns answering the question periodically; teachers get to decide the answering duration of each student. Through this activity, students are revealed to rely on their friends reasoning and work on it together, which will induce the students to make and revise their own mathematical conjectures.	All lesson plans except the first can incorporate this activity. Any of the questions in the presentations can be done through Round Robin. It can also be a filler activity (additional free time). This activity is better to be implemented further along the lesson due to the types of questions that the worksheets can cover. Teacher can manipulate any form of worksheet into a mock worksheet for Be the Teacher

	▪ Be the Teacher: teacher presents a sample worksheet of an imaginary student, where there are questions answered with mistakes along the way. Ask students to mark the paper and give feedbacks to the imaginary student, telling them what they did wrong and what to fix. With this activity, students get the chance to see mistakes from an outsider perspective, which helps them to learn from the mistakes without having to experience making the mistakes. Students will also form arguments as they mark the worksheet, which will take place concurrently with gaining the ability to develop and evaluate mathematical arguments.	activity. This activity is similar to Write, Pair, and Share, but with detachment of their seatmates, hence they can be more open in terms of sharing their mathematical reasoning.
Level 3 – Behaviors/ Transfers	▪ Students' coming worksheets and unit tests transcript; teachers can see if the students utilize the previous lessons in the coming worksheets and unit tests; as students who fail to learn will commit repeated mistakes and students who did learn will be able to work on the problem that possesses previous learning objectives. ▪ Post-test: students are given a problem that incorporates the previous learning objectives taught, so teachers can see if students are able to apply what they acquire, given that they have learned it. The post-test may also include a reasoning section where students are given the chance to explain how they come up with the solution. The post-test can also be done in a form of interview, where the teacher can observe the students' train of thought, given that it is the necessary approach the	This level of evaluation is done separately of the implementation of the instructional design. This level can only be done after the instructional design is fully implemented. For second hand data, teachers can use students coming worksheets and tests, and for first hand data, teachers can create a form of post-test to measure students' improvements.

	classroom needs. This post-test evaluation can serve to measure students' reasoning standards in terms of recognizing reasoning and proof as fundamental aspects of mathematics, and in selecting and using various types of mathematical reasoning.	

The author chose Model of Learning Evaluation by Kirkpatrick (1994) because it covers both formative and summative evaluation rigorously. The evaluation will further be separated into four levels, that is done in order. It will be adapted to the algebraic equations topic to further the relevance of the evaluation plan. However, since the Kirkpatrick's Model was not tailored specifically for teaching and learning mathematics, the evaluation of the reasoning standards coverage may not take place at every level of the evaluation. The evaluation plan is made to assist teachers in figuring out whether if the lesson plans have been customized to attend to the students' needs in the classroom and to improve the lesson plans at hand.

Teachers are suggested to follow all four levels, but if they deem one step to not be as necessary as the rest, it is also acceptable. This instructional design is created to make the lessons customizable and easy to follow, adapting to the varying needs of teachers to the vast types of classroom environment. Table 3 below describes examples of the activities corresponding to the levels in the evaluation plans, except level 4, as level 4 is done separately. The teachers can use these activities interchangeably in the lesson plans present in this instructional design, according to the potential implementation provided in Table 3.

CONCLUSION

In retrospect, through the development of the instructional design, the author realizes that there are always more layers to pay attention to in teaching a lesson. With that being said, it is challenging to decide upon which layer to focus on, because each seems to have their own importance. The instructional design has been made rigorous, but the author believes that it would be more meticulous if the author focused more on just one specific part of the lesson instead of attempting to broaden the coverage.

Teachers who implement this instructional design can adjust the learning activities accordingly to the needs of the classrooms through the evaluation plan and the activity design, which provides various activities ideas that teachers may implement in lieu of the previously suggested activities. If the activities are deemed to fit the teaching and learning activities, the teachers are also recommended to prolong the activity for upcoming lessons by adjusting the content while using the same learning activities. Furthermore, teachers can refer to the evaluation plan for various assessments and evaluation of the learning activities.

With regards to the structure of the lessons, it is possible to implicate more aspects of Understanding by Design, because the details of each lesson can always be made more intricate, with the assortment of strategies that is provided in Understanding by Design. In terms of the reasoning standards, there are many interpretations that can be implemented, in which the definition of mathematical reasoning can be broadened. The breadth of this instructional design can be increased through more synthesis of various reasoning standards.

To conclude this report of the construction of the instructional design, the author will describe the overall content present. The aim of this instructional design was to address the problems identified in an international private school institution that uses English as the language of instruction, which is inability to work on algebra due to prior misunderstanding and undeveloped mathematical reasoning in grade 8 students. The author addresses this problem through the involvement of mathematical reasoning in teaching algebraic equations.

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