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Uncovering Gaps in Deductive Geometry Thinking: Rasch-Based Evidence from Students' Work on Quadratic Functions

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Abstract

The deductive structure is fundamental in mathematics, but students often struggle to derive logical consequences when constructing proofs or engaging in the bridging process. This study analyzed students' ability to deduce logical relationships between properties in geometric thinking on quadratic functions. The participants were students (N=139) from the mathematics education program. The properties of quadratic functions and their interrelationships contained in the premises represented students' responses to the instrument. The responses were dichotomously coded based on the criteria of the carried-out deduction process and then evaluated using Rasch analysis. The findings revealed that, in general, the distribution of students' ability levels was below most levels of the carried-out deduction process. Although a group of students had already reached the high-ability category, the overall distribution was still dominated by low-ability levels. Many students continued to face serious challenges in reaching the final stage of the deduction process—advanced premise integration and deductive synthesis—since most of them established relationships among properties merely from the given information. There were indications that learning experiences during higher education contributed positively to the improvement of students' ability at the deduction level. The study recommends four steps to habituate students to the process of deduction.

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INTRODUCTION

The development of mathematics has established the deductive structure as a fundamental foundation of modern mathematical practice. Similarly, the recent rise of artificial intelligence is essentially grounded in deductive reasoning (Chowdhary, 2020). This highlights that the characteristics of deductive reasoning and its structural nature must be recognized as essential elements—ones that have long been embedded in mathematics education, both from the elementary level (Carreira et al., 2020), and in higher stages of learning (Miyazaki et al., 2017). The use of deductive reasoning structures is necessary for students in learning, and equally for teachers in teaching mathematics, alongside their efforts to strengthen mastery of mathematical content (Nurhidayah et al., 2019). The influence of this structure is reflected in how mathematics is taught, where instructional traditions often begin with formal definitions and fundamental axioms within a deductive system (Marja Van den Heuvel-Panhuizen, 2020), as can be seen in the dominance of such structures in textbooks (Doorman et al., 2020). Implementing this approach provides opportunities for students to formalize their knowledge by summarizing key concepts and ideas in a systematic order (Dinçer & Kaya, 2023), practicing proofs, building conviction, and developing mathematical creativity (Doorman et al., 2020). It also fosters deductive reasoning for understanding the logical structure of mathematics and the role of conceptual definitions within it (Bergman et al., 2024; Haj-Yahya et al., 2023; Siswono et al., 2020). As in geometry, generalization should not be drawn from the visual of figures, which represent only certain properties, because any change to a figure must be based on an axiom or a theorem so as not to produce contradictions in reasoning and the deductive structure (Karpuz & Atasoy, 2020). Moreover, the approach cultivates students' habits of thinking through logical sequences to make decisions based on valid information. The deductive structure thus serves as a vital foundation in mathematics and its learning, especially in the field of geometry.

However, students who reason deductively often face obstacles when constructing proofs or during the bridging process in proving. Deductive reasoning is the process of drawing specific conclusions based on general premises (Carreira et al., 2020; Chowdhary, 2020; Vargas & Stenning, 2020). Studi Miyazaki et al. (2017) found that Japanese students struggled to identify theorems for deriving intermediate statements or conclusions due to their limited understanding of universal instantiation and hypothetical syllogism. Similar difficulties have also been reported among Indonesian students (Siswono et al., 2020; Syahar et al., 2025). When generalizing patterns, students tend to perform only a few stages of reasoning without constructing proofs, providing arguments, or validating evidence (Syahar et al., 2025). In Portugal, Carreira et al. (2020) reported that students often gave short, unsystematic answers without employing complex inferential strategies when faced with deductive reasoning tasks in class. Jahnke and Krömer (2020) noted that even German mathematicians demonstrated distorted understandings of deductive structure, as they equated proof with justification and mistakenly assumed that proofs guarantee universal truth, whereas they in fact depend on the underlying theory or axioms. Vargas and Stenning (2020) further observed that undergraduate students in Portugal had difficulty controlling deductive reasoning processes, as they experienced anxiety in dealing with conjectures and uncertainty—stemming from a habitual reliance on mechanical procedures.

Beyond general mathematical reasoning, students' deductive reasoning in geometric thinking has shown similar patterns. More than 90% of Slovak university students still experience serious obstacles in deductive geometric reasoning—well below the expected competency level (Pavlovičová & Bočková, 2021). They struggle with the logical system of geometry and its formal proofs, revealing major issues in mastering advanced geometric thinking (Pavlovičová et al., 2022). Övez and Özdemir (2024) found that only a small proportion of Turkish students reached formal proof competence, while many remained confined to intuitive or procedural proofs that fail to meet the standards of mathematical deduction. Similarly, Scristia et al. (2025) reported that Indonesian students' development toward advanced levels of geometric thinking was hindered by the absence of classroom situations requiring formal deduction and strict generalization. Furthermore, a logical gap emerged because function graphs were often treated as "pictures" even though their processing was symbolic, which rarely synchronized the abstract symbolic properties with the relational geometric structure (Moore, 2021; Moreno-Armella, 2021).

In learning contexts, deduction can at least occur through two types of exercises: (1) deriving a logical statement solely from the information provided, or (2) deriving a logical statement from the given information combined with one's prior knowledge. Deduction of type (1) serves as a practice in applying basic logical operations—usually involving structured formal notation—while deduction of type (2) serves as an exercise in constructing a pathway toward discovering mathematical structures starting from simple and concrete ideas. These two processes differ in their execution. Based on the educational curriculum, Indonesian students more frequently perform deduction of type (1) during learning (Kodirun et al., 2025). When applying deduction of type (2), as reported by Lubis and Nasution (2017), students' understanding of logical operations did not significantly assist them. To perform deduction of this kind, students need the ability to retrieve and organize knowledge schemas within a coherent deductive chain. In geometry, deduction means working with abstract statements and drawing conclusions based on logic rather than intuition (Vieira & Cyrino, 2023). Success in this process largely depends on students' geometry content knowledge, problem-solving skills, and geometric reasoning (Llinares & Clemente, 2019). This process aligns with the van Hiele theoretical framework, which emphasizes reasoning about relationships between properties to produce a product of thought—a deductive system of properties (Van de Walle et al., 2017). In the context of functions, the visual form and algebraic structure became objects forming a deductive chain in the relational system through a bridging process (Wagner & Sharp, 2017). The present study adopts deduction of type (2) as the guiding pathway, based on Llinares dan Clemente's (2019) findings that highlight the importance of constructing relationships among geometric facts through known propositions, thereby supporting the bridging process in proof construction. Moreover, Miyazaki et al. (2017) described deduction of type (2) as the bridging process in constructing a proof with valid reasoning—a form of deductive reasoning derived from universal instantiation that generates a single proposition from a universal one.

Several studies related to deduction of type (2) in geometric contexts have been conducted, focusing on: the description of students' deductive thinking processes (Yudianto et al., 2019), the relationship between van Hiele levels, perception, and proof (Övez & Özdemir, 2024), and proof schemes in the transition from conjecture (Şen & Güler, 2022). The geometric content in those studies was largely based on Euclidean geometry. In this study, participants deduced premises from geometric content within the analytic domain— such as the quadratic function $f(x)=ax^2+bx+c$ with $f(x_1)=y=f(x_2)$ and $x_1+x_2=(-b)/a$, so that its axis of symmetry is $x=(-b)/2a$—which remains rarely explored. Scristia et al. (2025) investigated analytic geometry tasks designed to enhance formal deductive reasoning. Deduction of geometric content involving the analytic domain represents an adaptive step toward developing deductive levels of geometric thinking in this domain, which has traditionally focused on plane geometry derived from Euclidean axioms. Accordingly, this study aims to answer the research question: (RQ) How capable are university students in deducing relationships between properties in geometric thinking on quadratic functions?

RESEARCH METHODS

This quantitative study investigated university students' geometric thinking about quadratic functions at the deductive level. The participants were asked to organize a number of properties of quadratic functions based on their interrelationships so that they formed coherent statements within a deductive system.

Research data were collected using an instrument in the form of a task that required participants to construct premises. Participants were instructed to deduce based on the information provided, referring to specific characteristics of quadratic functions they already knew. This process guided them to establish relationships that could be deduced from the given information, resulting in a systematic reasoning structure. According to van Hiele's theory, an indication of geometric thinking at the deductive level is characterized by the relationship between properties as the object of thought and the deductive system of properties as the product of thought (Van de Walle et al., 2017). Hence, the properties of the quadratic function, as the objects of thought, were assumed to be interconnected and known to the participants, and the information was presented to them as a request to construct a premise. The instrument item presented to the participants was as follows: "Given: (1) $f(x)=ax^2+bx+c$ with $a\neq0$, and (2) a point (x, y) with abscissa x and ordinate y; Construct one premise based on (1) and (2) that is relevant to a characteristic or property of the quadratic function." The two pieces of information presented in the question—(1) and (2)—served as premises, so the participant's response represented a deduction result that was considered a premise in an argument, subsequently referred to as premise [3] for the purpose of data analysis.

The participants in this study consisted of 139 undergraduate students from a university in Jambi, Indonesia. They were students enrolled in the mathematics education program: first-year students (N=22) coded as group A, second-year students (N=48) coded as group B, and third-year students (N=69) coded as group C. Each group provided responses to the given instrument item.

After all responses were collected, we reviewed every answer submitted via Google Form. When a particular response appeared almost identical to another and was submitted within a short time interval, that response was deleted. Furthermore, the students' geometric thinking processes concerning properties and the relationships among those properties—as reflected in their written responses—were identified according to the response criteria. The following indicators defined the response criteria: (A1) involves a specific characteristic or property of a quadratic function, (A2) involves a relationship between a characteristic of the quadratic function and a premise, (A3) includes a representation of premise [3] based on the property of the quadratic function, (A4) involves a relationship between premise [1] and premise [2], (A5) involves a relationship between premise [1] and premise [3], (A6) involves a relationship between premise [2] and premise [3], (A7) involves a relationship among premises [1], [2], and [3], and, (A8) organizes all premises deductively into one complete statement. Criteria (A1) and (A2) were processes of identifying relationships among properties, and criteria (A3)–(A6) were processes of constructing inferential relationships among premises, whereas criteria (A7) and (A8) synthesized all premises into a coherent unity of arguments (Miyazaki et al., 2017), or, in deductive geometric thinking, were referred to as the deductive system of properties (Van de Walle et al., 2017).

For every student's response, a score of 1 was assigned to each criterion (A1)–(A8) that appeared (indicating that the student demonstrated that aspect of deduction), and a score of 0 was assigned to those that did not appear. Based on these response criteria, the person logit scale indicated the ability (able — unable), while the item logit scale represented the level of implementation (carried-out — not carried-out). An example of the coding procedure applied to participants' responses is presented in Table 1.

Table 1. Example of coding application on participant's response

Participant's response	Code name	Coding for item process criteria							
		A1	A2	A3	A4	A5	A6	A7	A8
a. If $a>0$ then $f(x)$ open upward; if $a<0$ then $f(x)$ opens downward	124C	1	1	1	0	1	0	0	0
b. Given that the quadratic function $f(x)=ax^2+bx+c$. If the point (x,y) lies on $f(x)$, then $y=ax^2+bx+c$.	132A	0	0	0	1	0	0	0	0

In Table 1, based on the statement "If $a>0$ then $f(x)$ open upward; if $a<0$ then $f(x)$ opens downward" this shows a property or characteristic of the quadratic function presented by respondent 124C, namely that a quadratic function has the characteristic of its graph opening either upward or downward, thereby fulfilling criterion A1, which was coded with the value 1. The code 124C indicates a third-year mathematics education student with response number 124. Respondent 124C also incorporated the relationship between the property of the quadratic function (its opening direction) into the statement using the "if … then …" formation, so criteria A2 and A3 were also coded as 1. The statement additionally involved a relationship between premise [1] and premise [3], since it connected the coefficient a of x^2 in $f(x)$ with the graph's opening direction; therefore, criterion A5 was coded as 1.

The data from each student, which had been coded as 1 or 0, were then analyzed using the Rasch model method. This method scales responses in logit units (Bond & Fox, 2015), examining how each student's ability and the carried-out items of the deductive process reflected the relationships among the properties of quadratic functions according to the response criteria. As conducted by Seah and Horne (2020) in their study identifying eight thinking zones to develop geometric reasoning, this method validated the hierarchical structure of criteria (A1)–(A8) as a process model in deductive geometric thinking based on the conformity of response patterns.

RESULTS AND DISCUSSION

Based on the results of the data analysis, Table 2 presents a summary of Rasch analysis statistics, grouped according to person and item.

According to Table 2, the mean logit values for person and item processes were –1.41 and 0.93, respectively. These values represent the logit scale range of ability level and the level of carried-out deductive processes, indicating that, in general, students' ability levels were below the level of the carried-out deductive processes. This finding suggests that many students encountered challenges in certain stages of deducing the relationships among the properties of quadratic functions. Furthermore, the person separation index was 1.98 with a reliability of 0.80, meaning that the instrument could distinguish respondents into two distinct ability levels with good consistency. The internal reliability of the instrument, represented by Cronbach's Alpha (KR-20), was 0.82, indicating satisfactory consistency in measuring respondents' ability. The global goodness-of-fit test yielded a Chi-Square (χ^2) value of 411.97 with a significance of $p<0.01$, suggesting that the data fit the Rasch model well. From the perspective of the process items, the mean carried-out level was 0.93 logit, with a standard deviation of 3.97, signifying a wide variation in carried-out processes—from very easy to very difficult to carry out. The item separation index was 4.82, with a reliability of 0.96, indicating that the instrument could classify the carried-out process items into seven distinct levels of carried-out difficulty with very high consistency. Overall, these results demonstrate that the instrument had excellent discriminating power for process items and high reliability for measuring the carried-out levels of deductive processes, as well as good reliability for measuring respondents' abilities. This confirms that the instrument can be reliably used to evaluate respondents' abilities across a sufficient range of carried-out deductive processes.

Table 2. Summary of person and item statistics

	Person	Item Process
N	139	8
Measure (logit)		
Mean	-1.41	0.93
Standard deviation, SD	3.61	3.97
Standard Error, SE	0.31	0.53
Separation	1.98	4.82
Reliability	0.80	0.96
Cronbach's Alpha	0.82	
Raw variance	79,8%	
Chi-Square (χ^2)	411.97*	
*$p<0.01$		

Students' Ability Levels

Based on the data analysis results, Table 3 presents the distribution of students according to their respective ability levels.

In the high-ability category, Table 3 shows three groups with different logit values. Ten students had the highest ability level with a logit of 7.08; they were able to complete the process of deducing relationships among the properties of quadratic functions to form a deductive system of properties of quadratic functions. In addition, four respondents were at logit 4.53, and twenty-five respondents were at logit 1.85. In total, 39 students (28.1%) fell into the high-ability category. The probabilities of students from the first, second, and third years belonging to this category were 0.091 (N=2/22), 0.333 (N=16/48), and 0.304 (N=21/69), respectively.

Table 3. Distribution of participants by ability level

Level	Logit Interval	Measure (Logit)	N	Name Code
High	0.66 ≤ LVI ≤ 7.08	7.08	10	009B, 013B, 017B, 020B, 035B, 076C, 078C, 098C, 101C, 103C
		4.53	4	067C, 100C, 108C, 125C
		1.85	25	003B, 006B, 016B, 021B, 028B, 029B, 032B, 039B, 040B, 043B, 044B, 054A, 056A, 061C, 062C, 068C, 082C, 084C, 085C, 088C, 090C, 114C, 116C, 117C, 126C
Low	−5.76 ≤ LVI < 0.66	0.28	10	010B, 037B, 038B, 063C, 070C, 079C, 097C, 099C, 105C, 124C
		–0.94	8	002B, 025B, 026B, 036B, 083C, 109C, 113C, 120C
		–2.37	21	005B, 008B, 033B, 052A, 058A, 059A, 066C, 071C, 075C, 080C, 081C, 086C, 091C, 096C, 110C, 112C, 115C, 122C, 127A, 131A, 134C
		–4.07	44	001B 004B 007B 011B 014B 015B 019B 022B 023B 031B 041B 042B 045B 046B 047B 048A 049A 050A 053A 057A 060C 074C 077C 087C 089C 092C 093C 094C 095C 102C 104C 106C 107C 111C 121C 123C 128A 130A 132A 135A 136A 137A 138A 139B
		–5.76	17	012B, 018B, 024B, 027B, 030B, 034B, 051A, 055A, 064C, 065C, 069C, 072C, 073C, 118C, 119C, 129A, 133C

In the low-ability category, the distribution of respondents was more dispersed, consisting of five groups with different logit values. Ten respondents were at logit 0.28, eight at logit –0.94, and twenty-one at logit –2.37. The largest group was at logit –4.07 with forty-four respondents, while seventeen respondents were at the lowest ability level with logit –5.76. Altogether, 100 students (71.9%) belonged to the low-ability category. The probabilities of students from the first, second, and third years belonging to the low-ability category were 0.909 (N=20/22), 0.667 (N=32/48), and 0.696 (N=48/69), respectively.

This distribution pattern indicates that the majority of respondents (71.9%) were still in the low-ability category. Not only do international studies (e.g. Övez & Özdemir, 2024; Pavlovičová et al., 2022; Pavlovičová & Bočková, 2021; Scristia et al., 2025; Siswono et al., 2020; Syahar et al., 2025) reveal a similar tendency, but this study also confirms the dominance of low-level deductive ability. Furthermore, most students (25 out of 39) within the high-ability group were still at a logit of approximately 1.85, which is the high-ability level closest to the low-ability range.

Although some students had achieved the high-ability category, the overall distribution remained dominated by low-ability levels.

These findings highlight that most students still require intensive learning support to transition from the low to the high category to enhance their geometric thinking ability at the deductive level, for instance, through stages aligned with the sequence of process items identified in this study. This also reinforces that deductive reasoning and its structural nature should be recognized as essential elements (Carreira et al., 2020; Miyazaki et al., 2017), which still need to be strengthened in educational practice (Nurhidayah et al., 2019). The findings of this study are consistent with Scristia et al. (2025) who found that the development of Indonesian students' advanced geometric thinking requires classroom situations that foster formal deduction and strict generalization.

On the other hand, second-year students had the highest probability of being in the high-ability category, whereas first-year students had the greatest probability of belonging to the low-ability category. This suggests that more senior students tended to have a better ability distribution than first-year students. The difference may be attributed to variations in student input quality, learning experiences in higher education, or curriculum and pedagogical factors across cohorts. It may also be interpreted that the longer students study at university, the greater their chances of achieving a higher ability category, although the rate of transition from low to high ability remains relatively slow. In other words, there is an indication that learning experiences in higher education contribute positively to the improvement of students' deductive-level abilities.

Levels of Carried-Out Deductive Processes

In this study, eight process criteria were defined to generate a deductive statement. The distribution of carried-out levels for each process item is presented in Table 4.

Table 4. Level of carried-out deductive processes

Level	Rentang Logit	Measure (Logit)	Infit MNSQ	Outfit MNSQ	PT-Mea Corr.	Item Code
Very difficult	5.77 ≤ LVI ≤ 7.46	7.46	-	-	0.35	A8
		6.17	0.87	0.05	0.67	A7
Difficult	4.09 ≤ LVI < 5.77	-	-	-	-	-
Moderately difficult	2.40 ≤ LVI < 4.09	2.71	1.35	3.13	0.63	A6
Moderate	0.72 ≤ LVI < 2.40	-	-	-	-	-
Moderately easy	−0.97 ≤ LVI < 0.72	–0.11	0.35	0.16	0.86	A5
		–0.21	0.44	0.24	0.85	A3
		–0.73	0.69	2.01	0.81	A2
Easy	−2.65 ≤ LVI < −0.97	-	-	-	-	-
Very easy	−4.34 ≤ LVI < −2.65	–3.51	0.66	0.40	0.68	A1
		–4.34	1.54	9.90	0.40	A4

According to Table 4, the Rasch analysis results for the eight process items indicated a wide variation in carried-out levels (–4.34 to 7.46). Several processes that were difficult to carry out were represented by positive logit measures, such as items A6, A7, and A8. The most difficult or least frequently carried-out process among students was item A8, which involved organizing all premises deductively into a single statement. An example of a student response that met category A7 was: "If (x,y) satisfies $f(x)=ax^2+bx+c$, then the point (x,y) lies on the graph of the function". Previous studies have revealed that students often attempted to organize concepts but had not yet achieved mature conceptual integration (Alghadari et al., 2022), their understanding remained partial and non-integrative (Alghadari et al., 2019; Hutajulu et al., 2022; Noor & Alghadari, 2021). In contrast, other items—A4, A1, A2, A3, and A5—were easier to carry out, with A4 (relating $f(x)=ax^2+bx+c$ to the point (x,y)) being the most frequently and easily carried-out process among students.

Regarding model fit, most process items fell within the acceptable range of infit and outfit mean square values (0.5–1.5), for instance, A7 (infit 0.87; outfit 0.05) and A1 (infit 0.66; outfit 0.40), confirming their consistency with the Rasch model. However, some carried-out items showed deviations, such as A6 (infit 1.35; outfit 3.13) and A2 (infit 0.69; outfit 2.01), indicating anomalies among certain respondents. Some students performed these processes even though their probability of doing so was relatively low, likely due to difficulties in integrating logical components among the premises. Students who were expected to relate the features of quadratic functions to specific details in the premises sometimes failed to show such indications, or instead invoked other properties not referenced in the carried-out item process. This instability likely stemmed from the transition from facts to premises, consistent with Lubis and Nasution (2017), who stated that students' understanding of logical operations did not substantially assist them.

Other deviations were detected in items A5 (0.35; 0.16) and A3 (0.44; 0.24), which had mean square values that were too low, suggesting that these carried-out processes appeared as easily predictable deduction stages. This occurred because students could directly use the given information in the instrument as components of the premise, without requiring complex logical relations—making these steps appear mechanical. The characteristics of A3 and A5 align with the curriculum context in which students are more accustomed to practicing basic logical operations using structured formal notation (Kodirun et al., 2025), often without deep logical reflection (Vargas & Stenning, 2020), or without engaging complex inferential strategies (Carreira et al., 2020). Item A4 (outfit 9.90) exhibited a high carried-out level, performed by students of both low and high ability categories. Because of this small variation, the item was less informative for distinguishing student ability based on the carried-out deductive processes. Moreover, this criterion did not require participants to engage in high-level relational reasoning.

The PT-Measure Correlation, representing the correlation between carried-out process items and students' abilities, indicated that all items had values not lower than 0.30, suggesting good contribution since each carried-out process item could still distinguish students by their ability. With high correlations and near-ideal MNSQ values, several carried-out process items—particularly A2, A3, and A5—proved most effective in distinguishing among student ability categories.

Overall, the analysis results indicate that most process items functioned well in measuring the carried-out deductive processes in geometric thinking on quadratic functions, especially A7 and A1, which showed high conformity with the Rasch model. Hence, the research instrument demonstrated excellent quality in differentiating levels of carried-out process items in generating a deductive system of properties.

This study revealed a sequence of deductive processes categorized into four levels of carried-out performance. The sequence reflected the hierarchy of cognitive complexity in mathematical thinking, which in this study specifically related to quadratic functions and their premise relations. The progression moved from basic and direct steps toward more complex and synthetic ones. According to the sequence, the four levels of the deductive process were as follows: the first level included processes A4 and A1; the second level included A2, A3, and A5; the third level consisted only of A6; and the fourth level included A7 and A8. Each step represented an essential action within the deductive process. After examining each step and its associated process items, these were further reduced into the following stages: (a) analysis of basic quadratic function properties, (b) integration of quadratic functions and premises, (c) integration of extended premises, and (d) deductive synthesis. These four stages—(a) through (d)—constitute the recommended procedures for student learning, especially when deducing mathematical properties and the relationships among them. The stages reflect that deduction occurs through the integration of knowledge within a complex logical chain, described by Fernández-León dan Gavilán-Izquierdo (2022) as the dynamic interaction between using and creating mathematics. Consistent with Doorman et al. (2020), the deductive process fosters creative and systematic thinking. These stages can also be adapted to other deductive contexts as appropriate. Thus, deriving a logical statement based on presented information combined with students' prior knowledge serves as an exercise for constructing pathways toward discovering mathematical structures—beginning with simple and concrete concepts—and can be realized through these four stages.

CONCLUSION

Based on the findings of the study, the following conclusions were drawn. The distribution of students' ability levels was generally below most levels of carried-out deductive processes. Students were able to complete processes that did not require complex logical relations and tended to be mechanical in nature. On the other hand, many students still faced challenges in certain processes of deducing relationships among the properties of quadratic functions, particularly in the stages of advanced premise integration and deductive synthesis. Although the overall distribution was still dominated by students with low ability levels, and the rate of transition from low to high ability categories remained relatively slow, there was an indication that learning experiences during university study contributed positively to improving students' ability at the deduction level. In addition, there was a considerable variation in the deductive processes, ranging from those that were very easy to those that were very difficult for students to perform. This variation eventually produced four levels of deductive process implementation, which became the basis for developing the recommended procedure for student learning, particularly when deducing mathematical properties and the relationships among those properties.

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